Optimizing Tail Latency in the Light-Tailed M/G/k

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Cornell University

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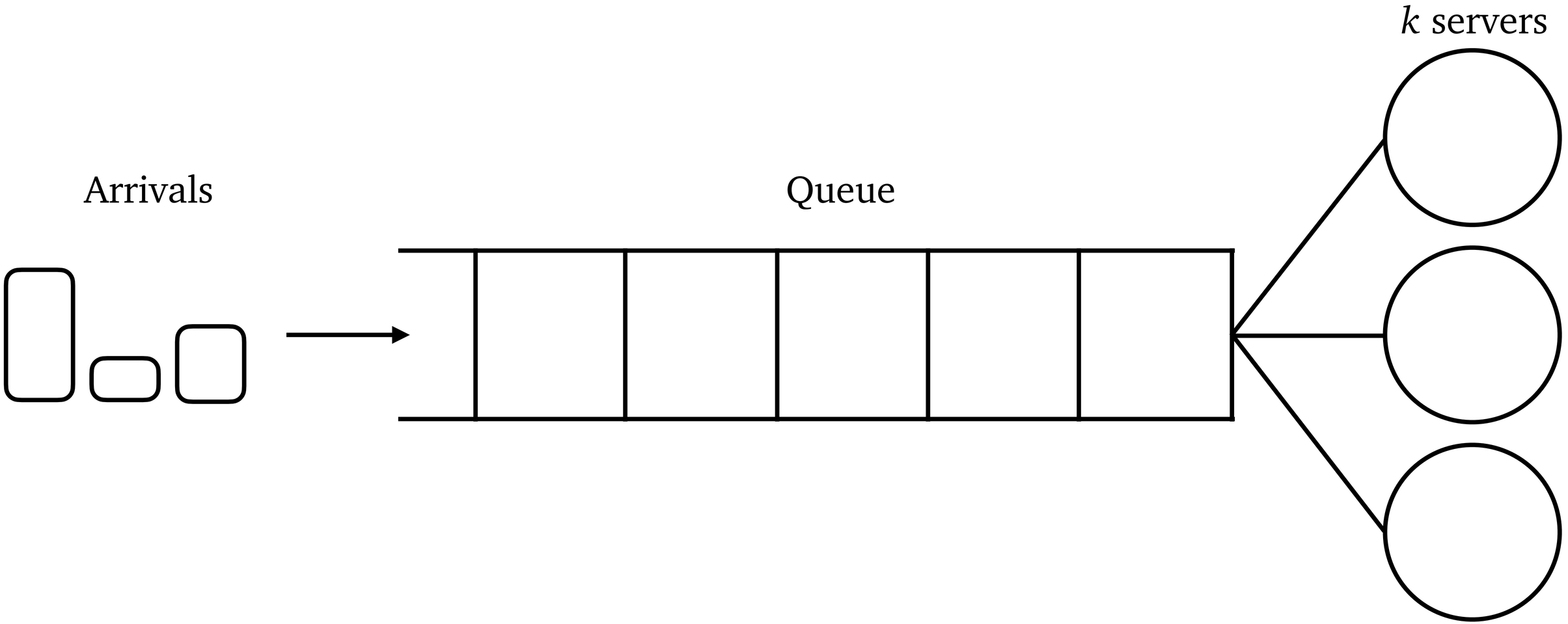
The light-tailed M/G/k



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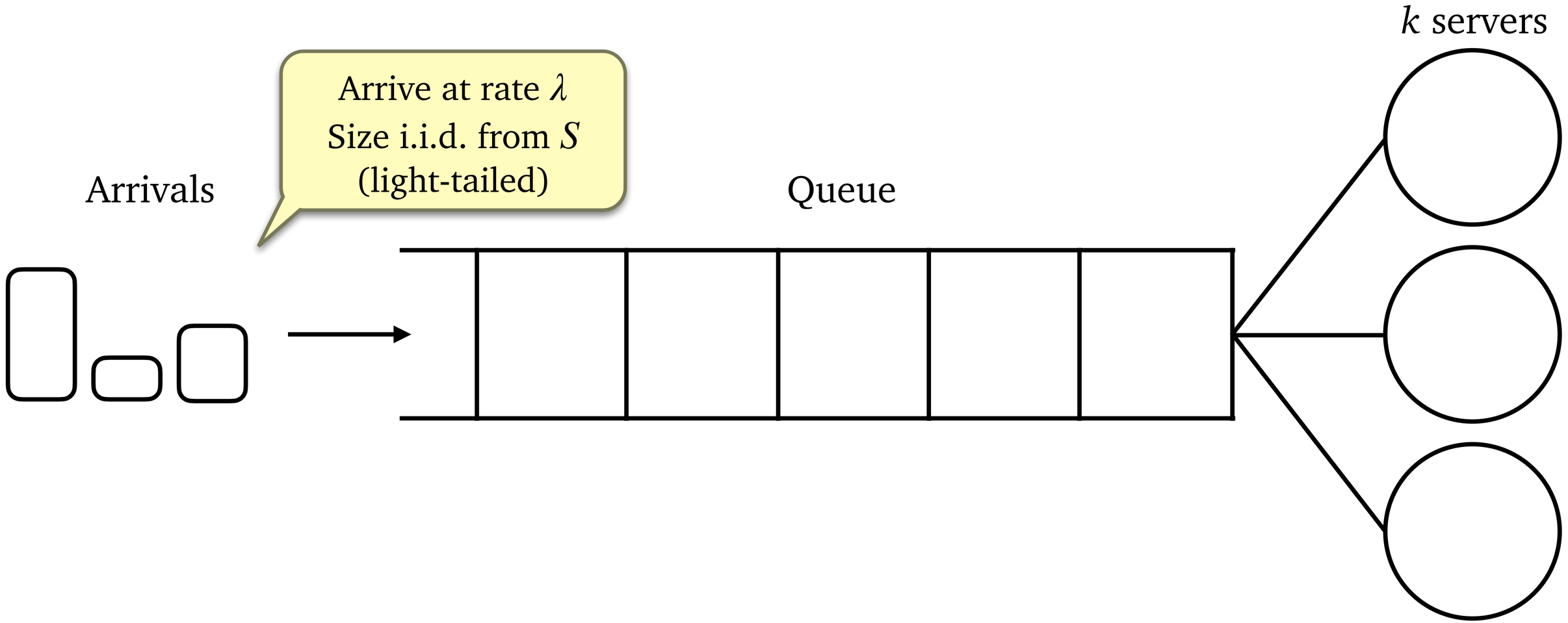

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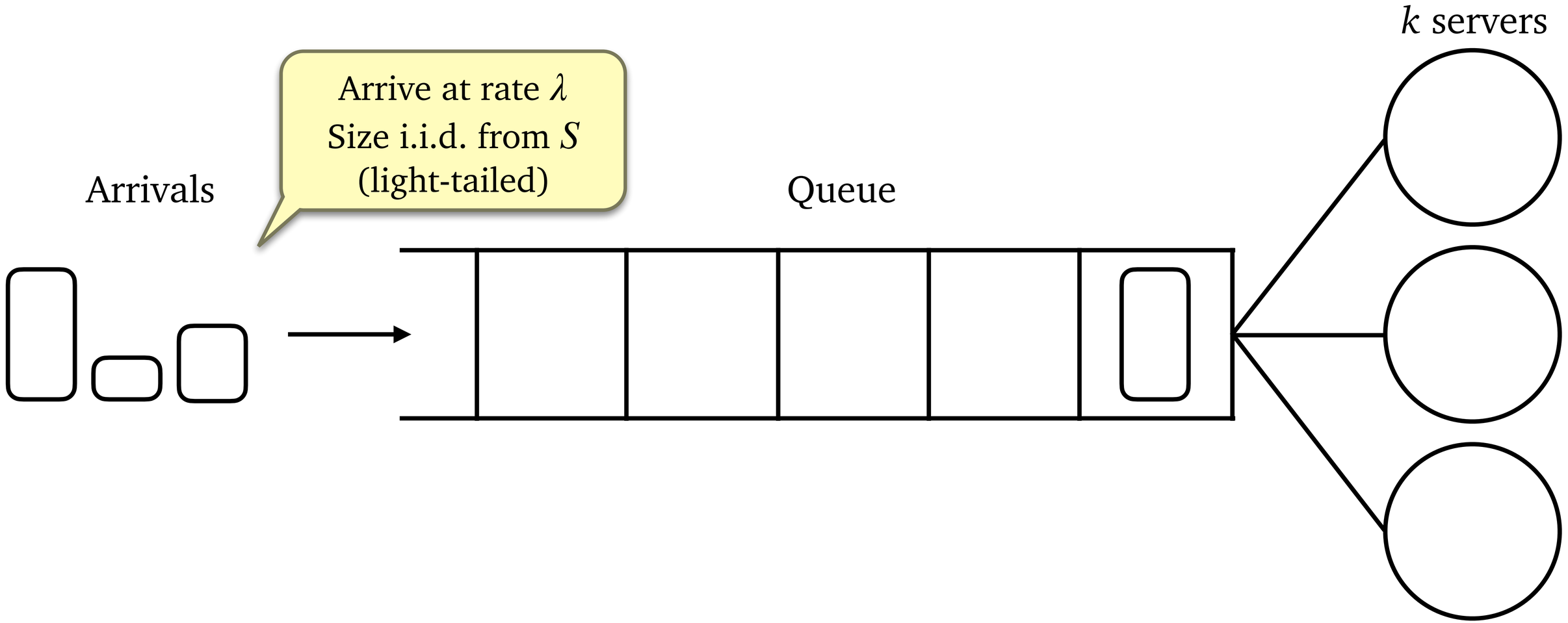

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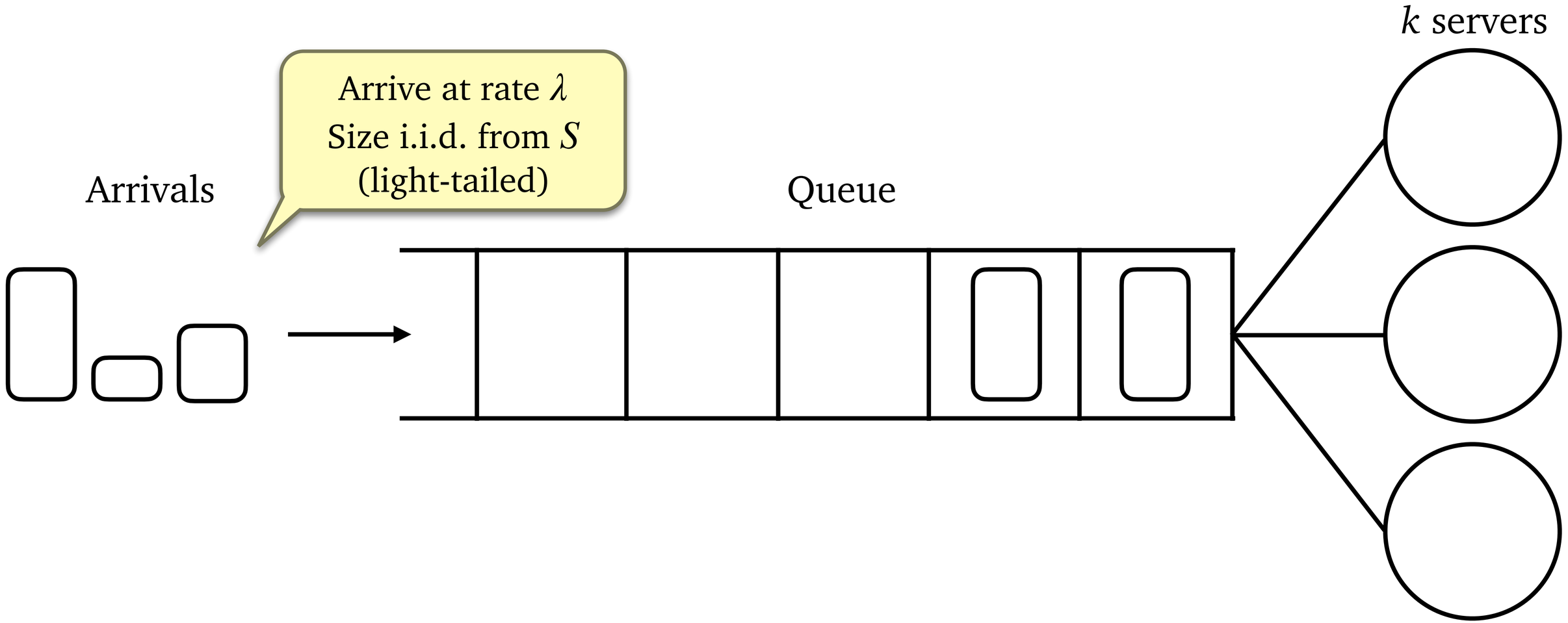

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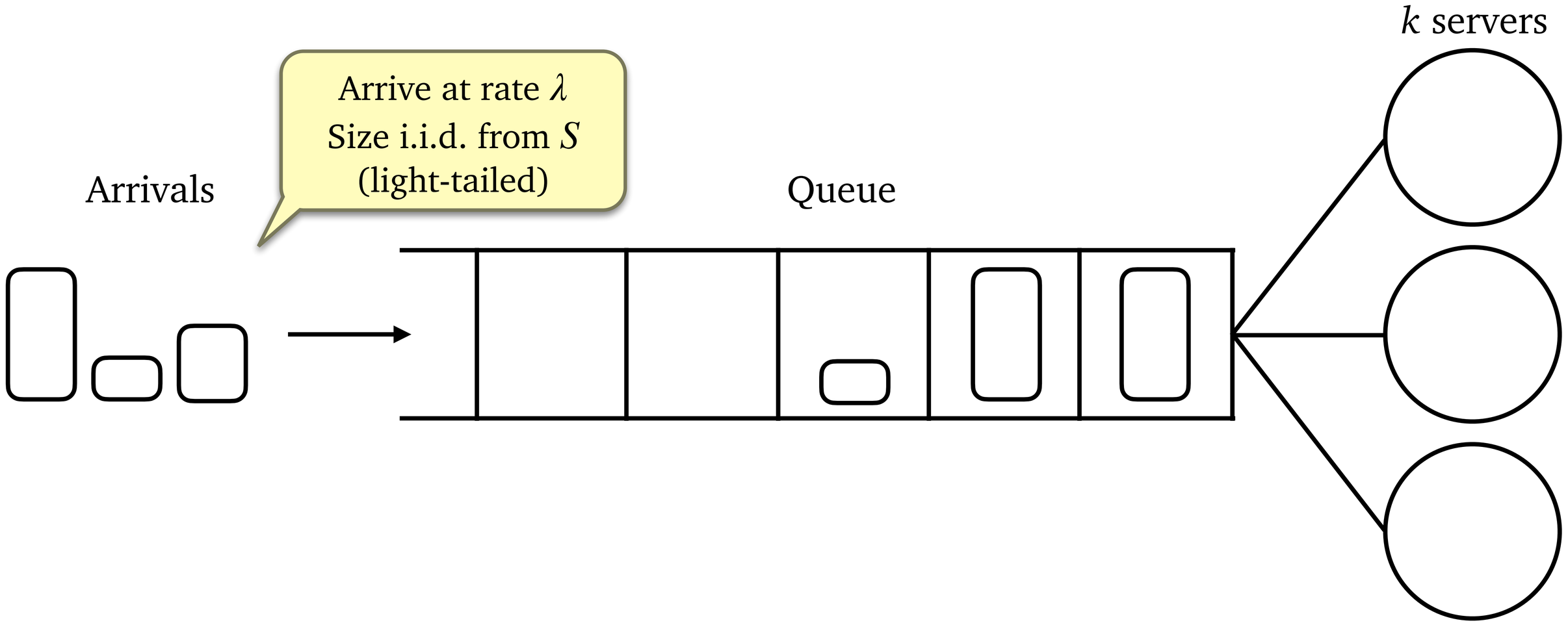

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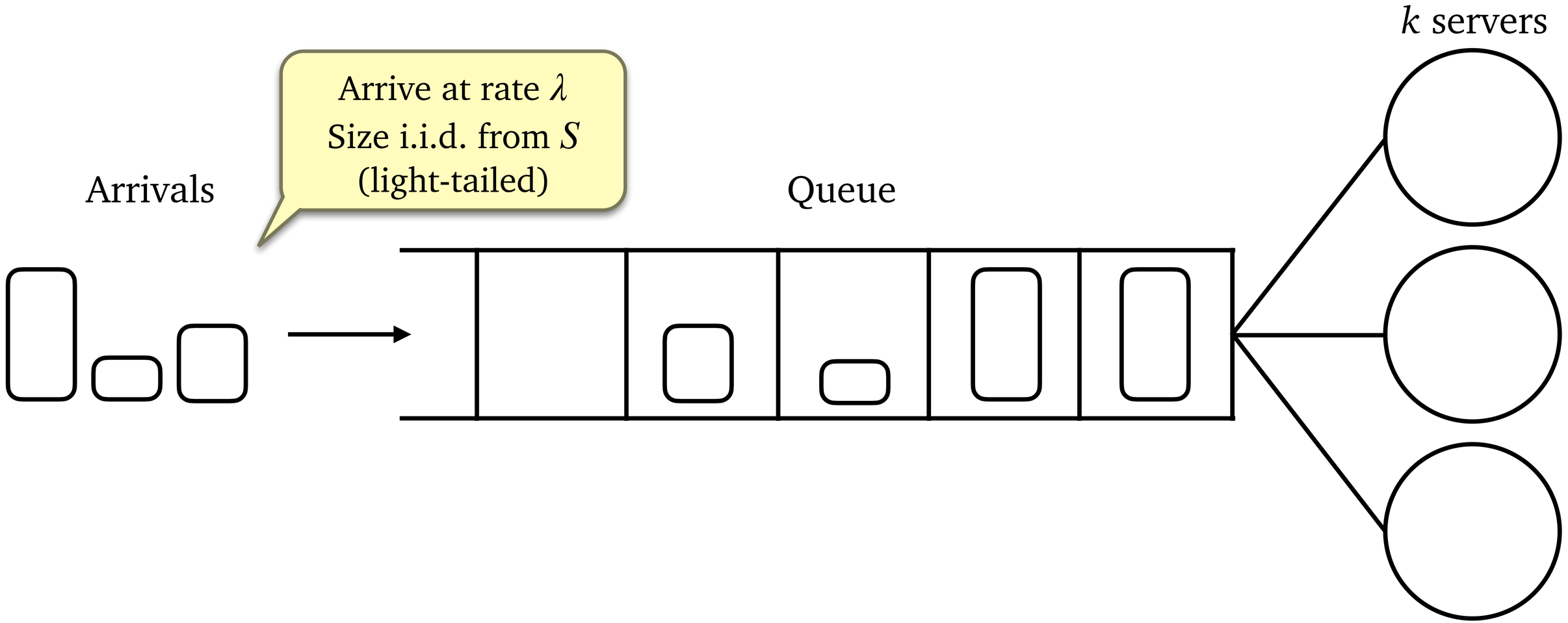

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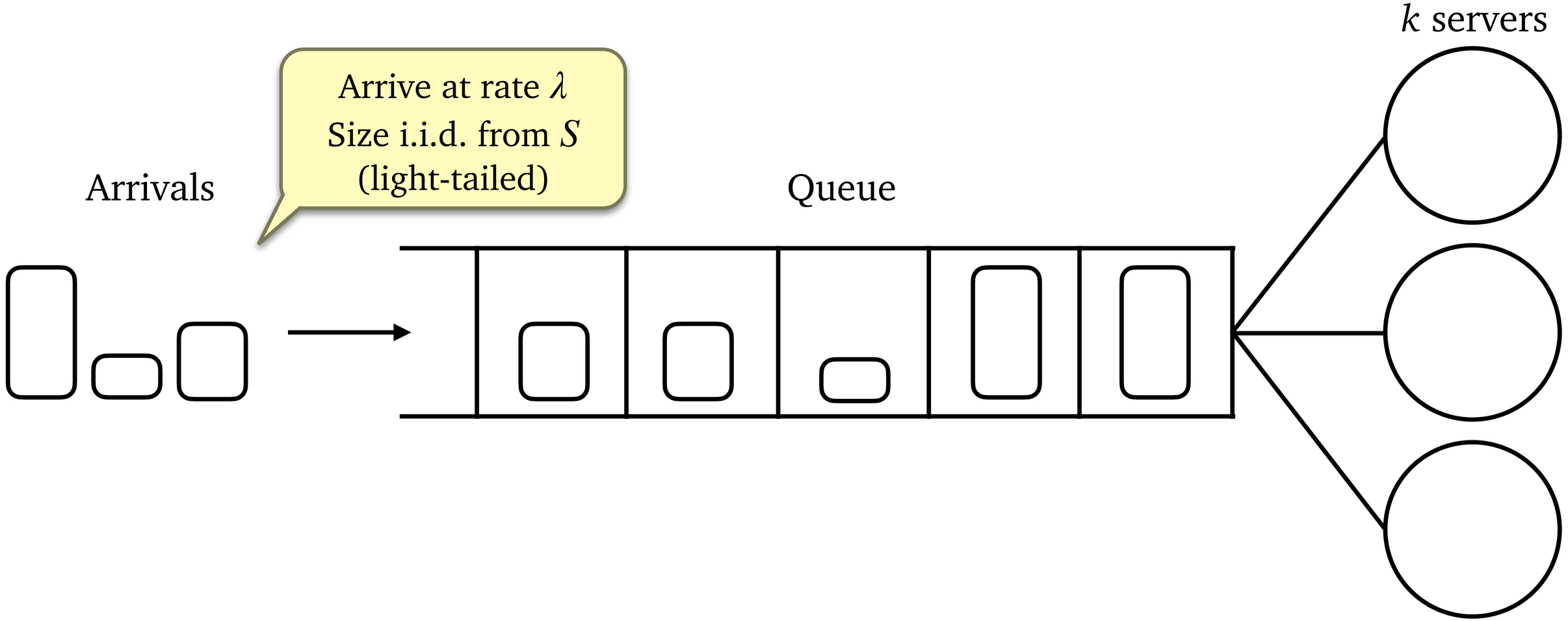

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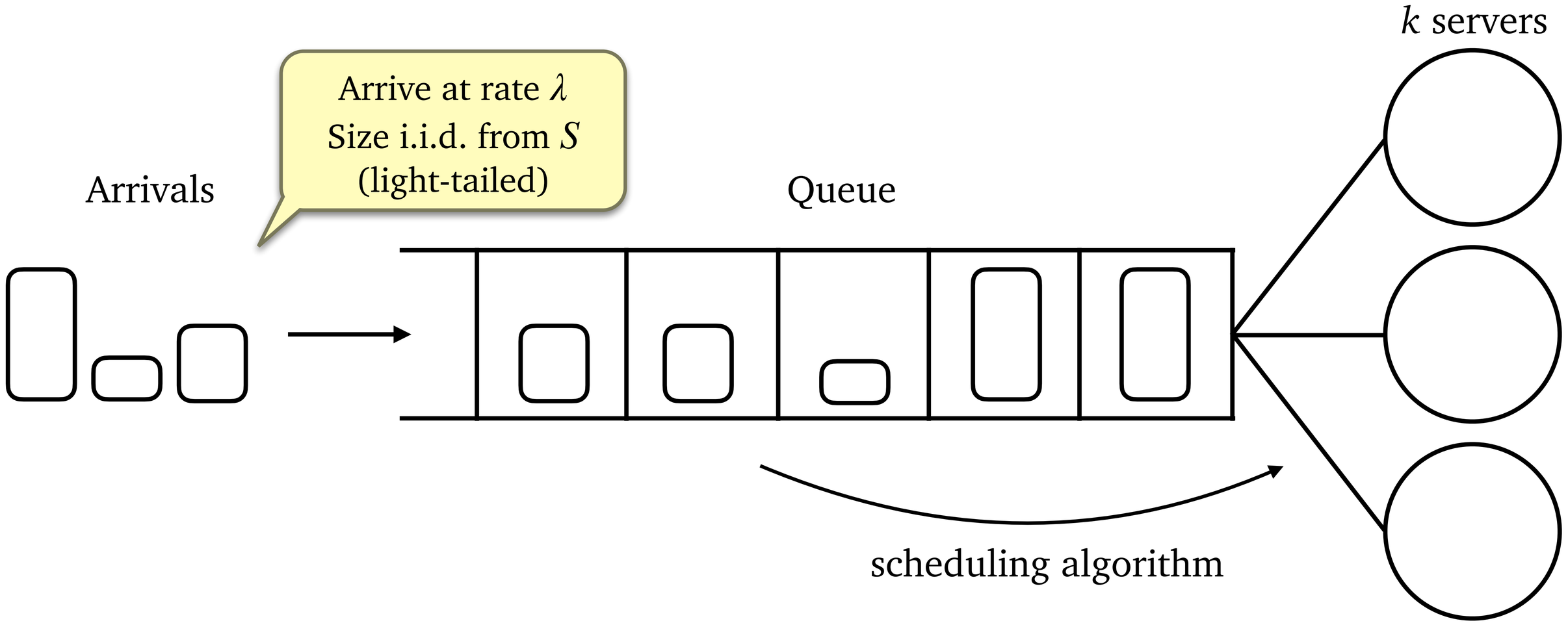

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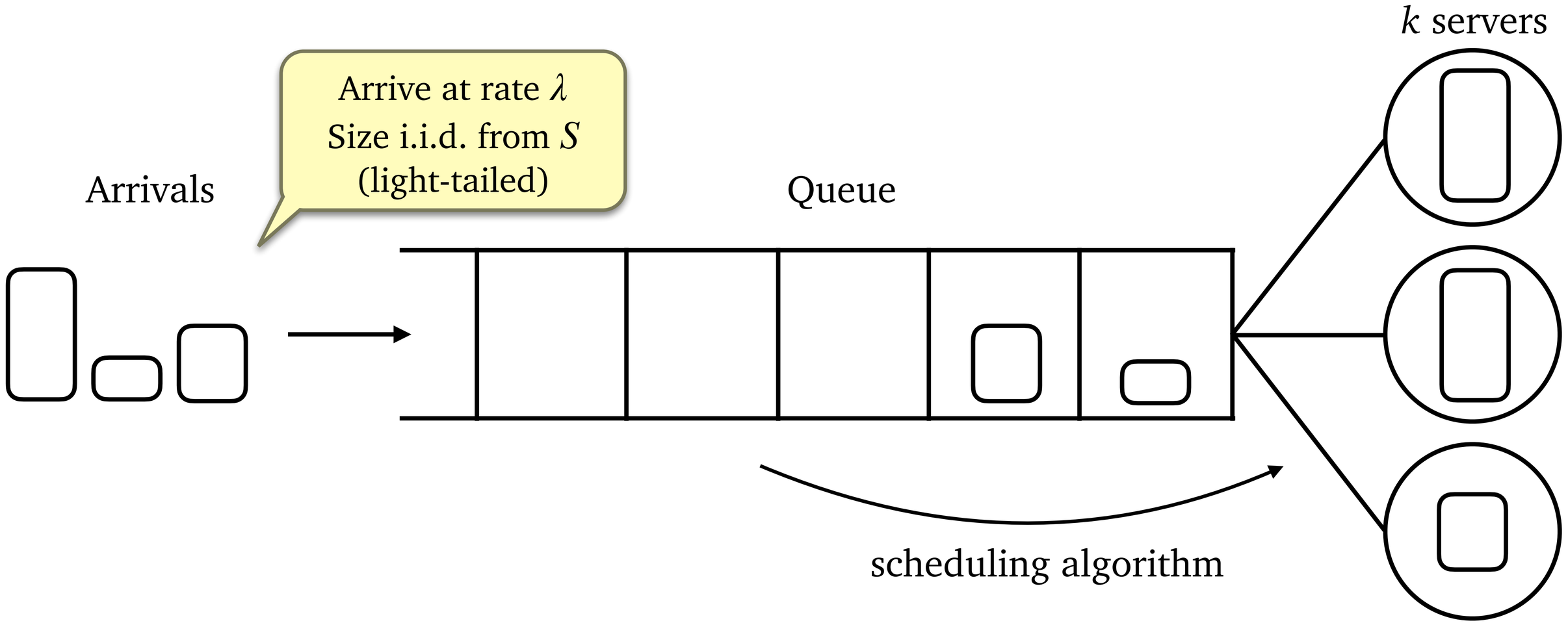

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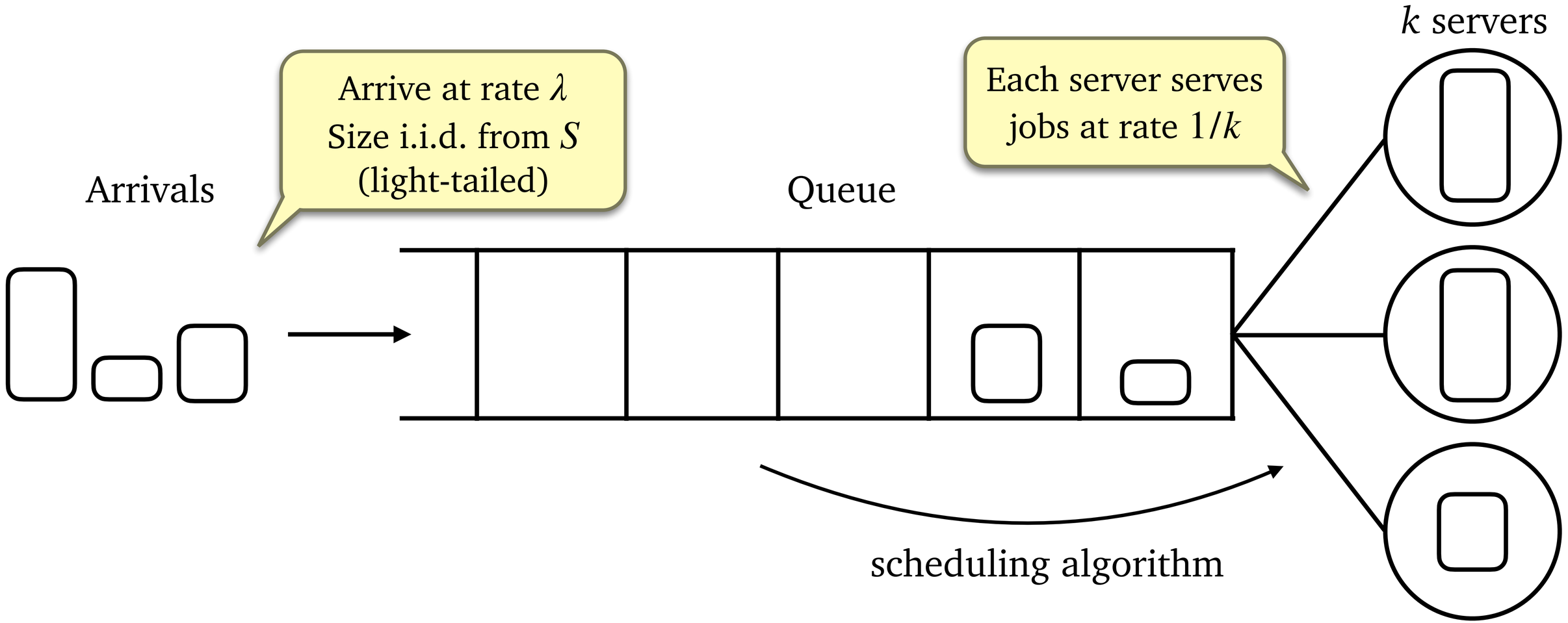

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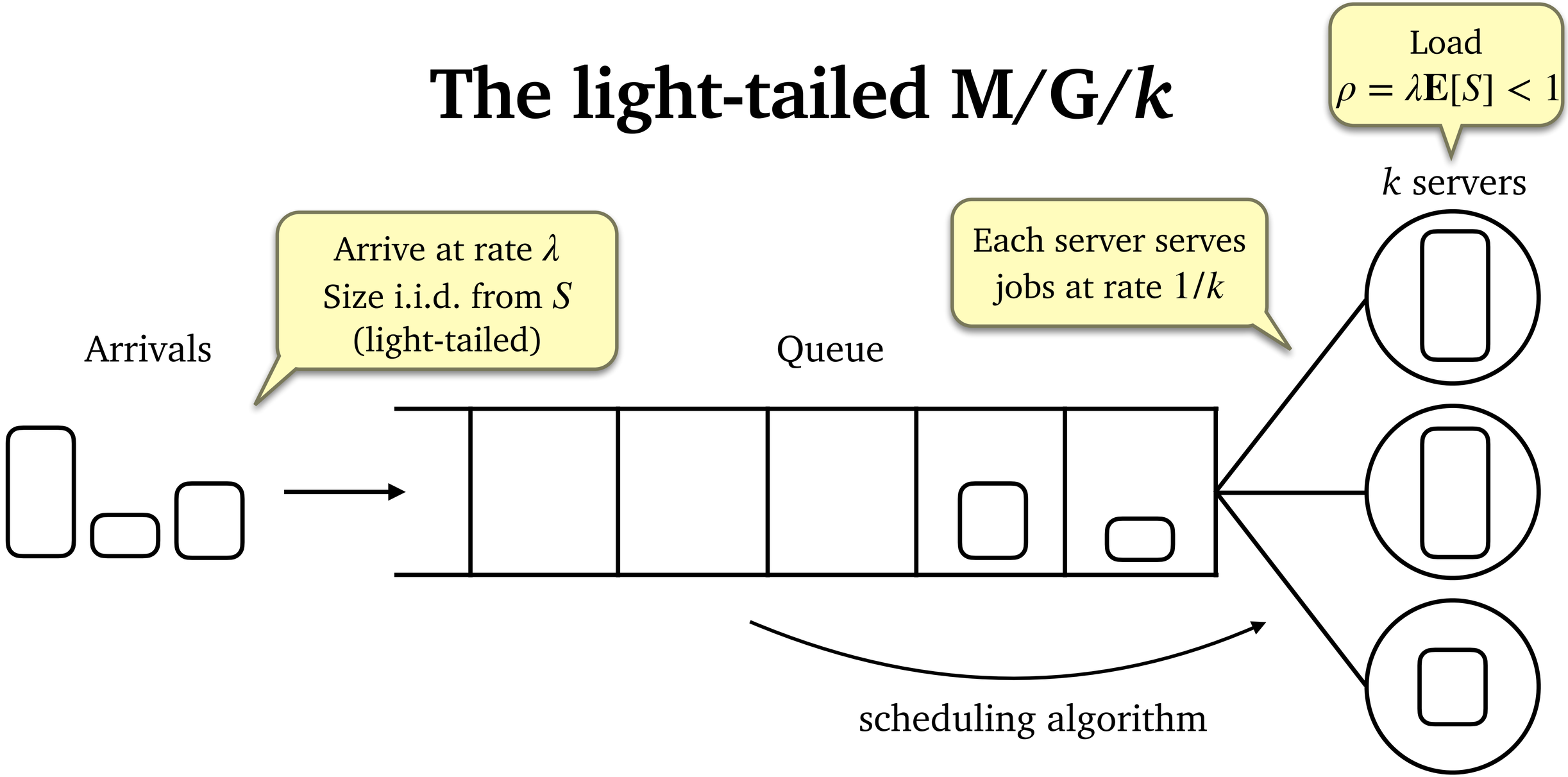

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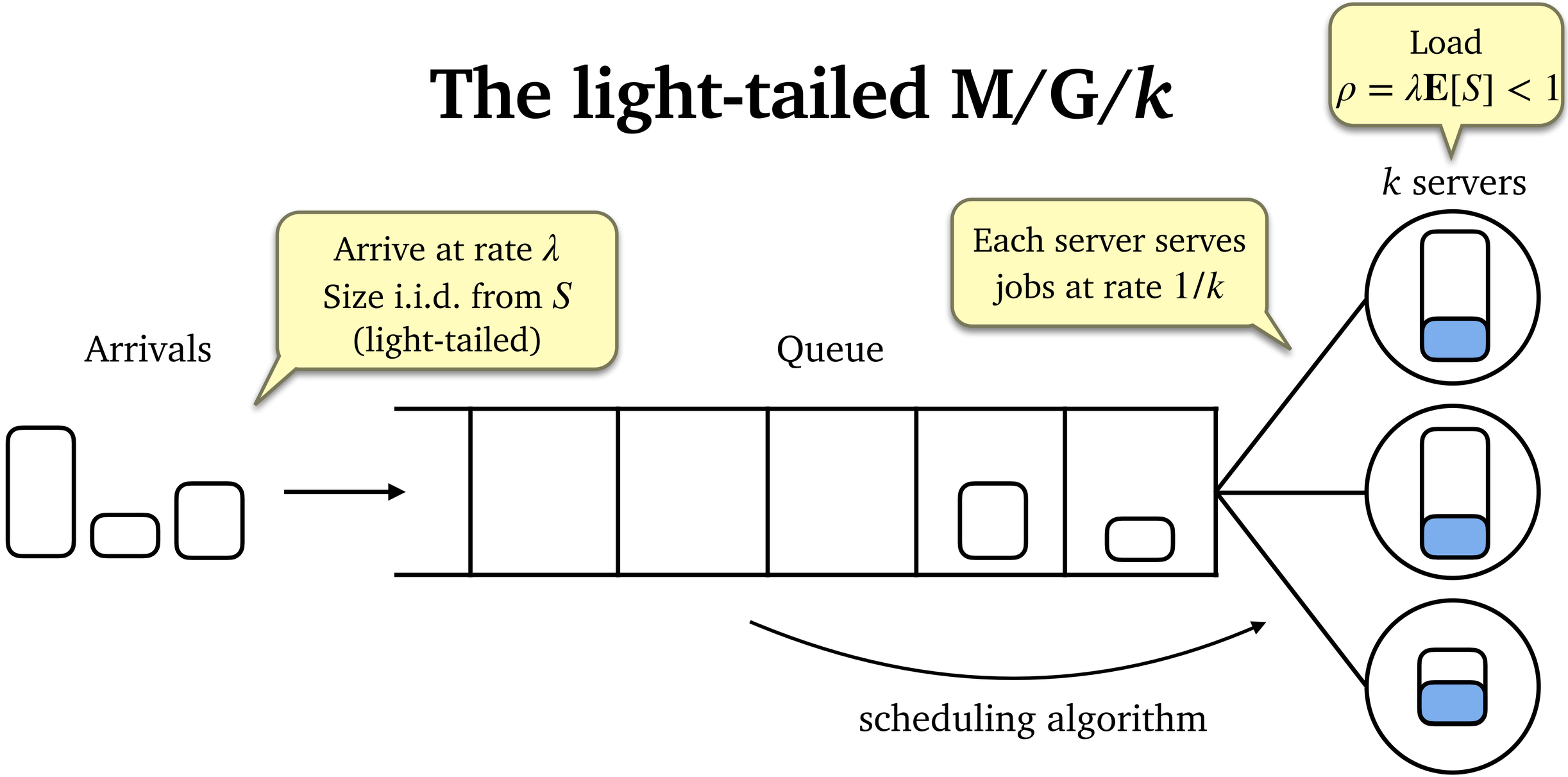

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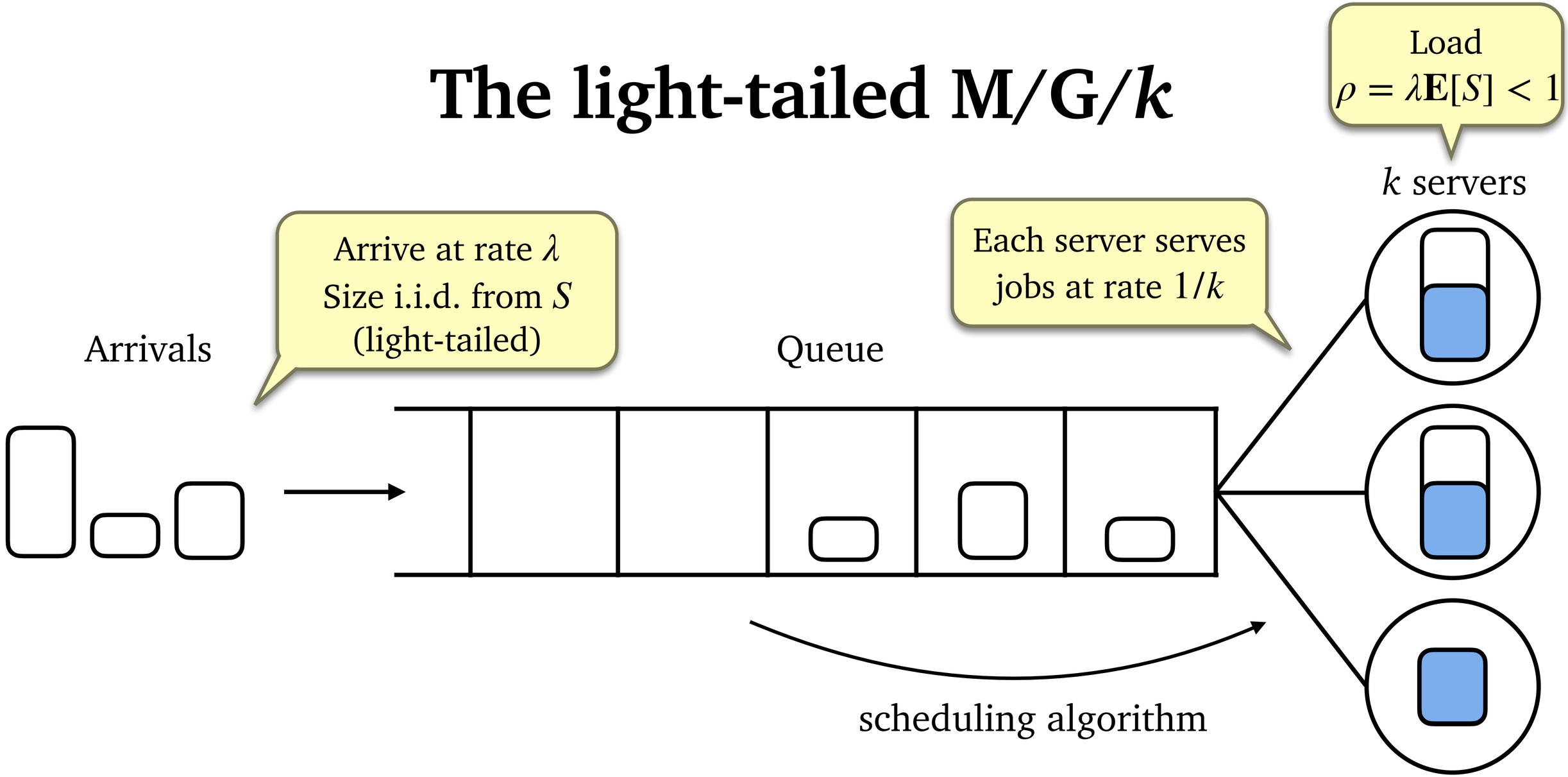

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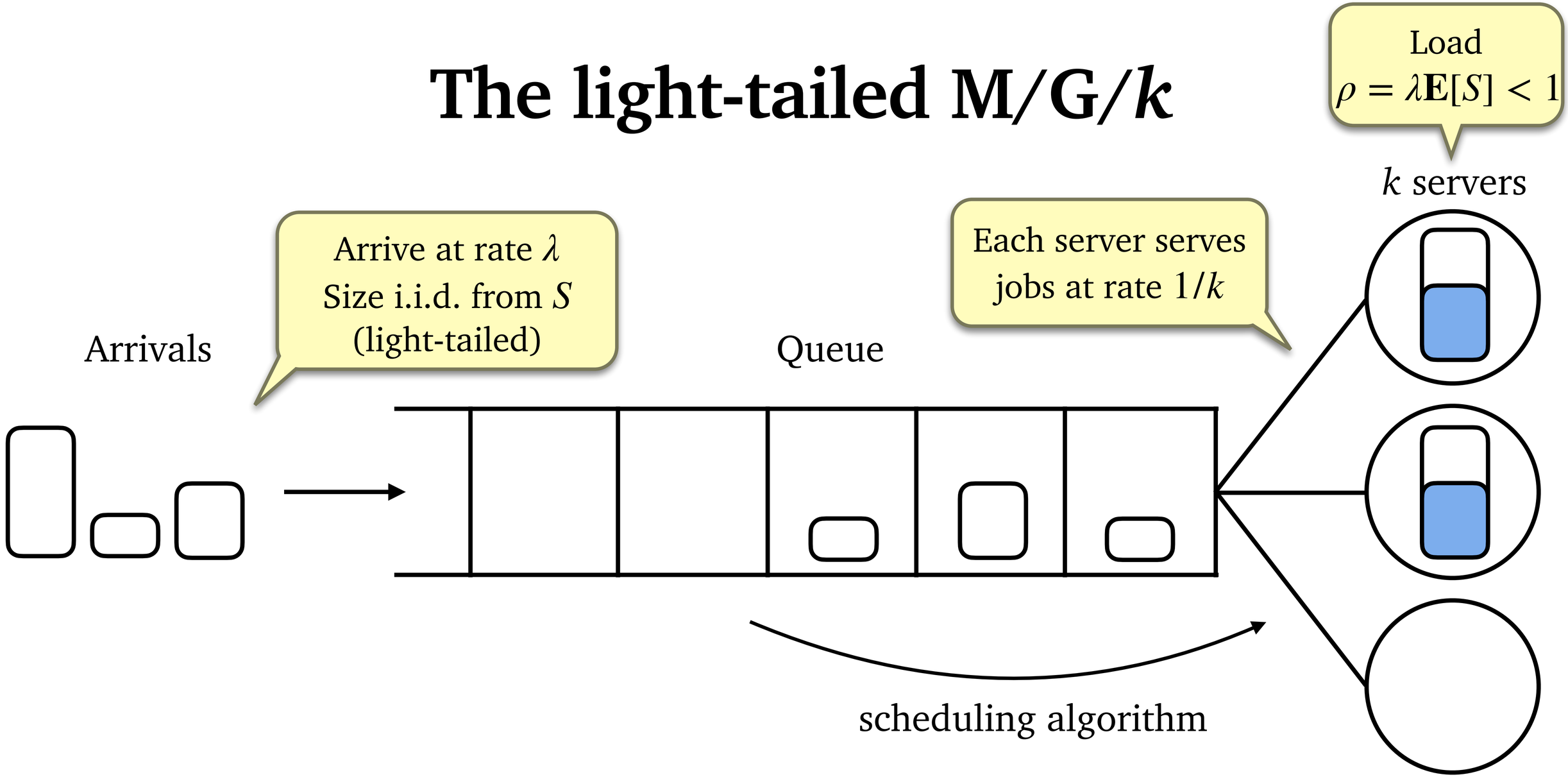

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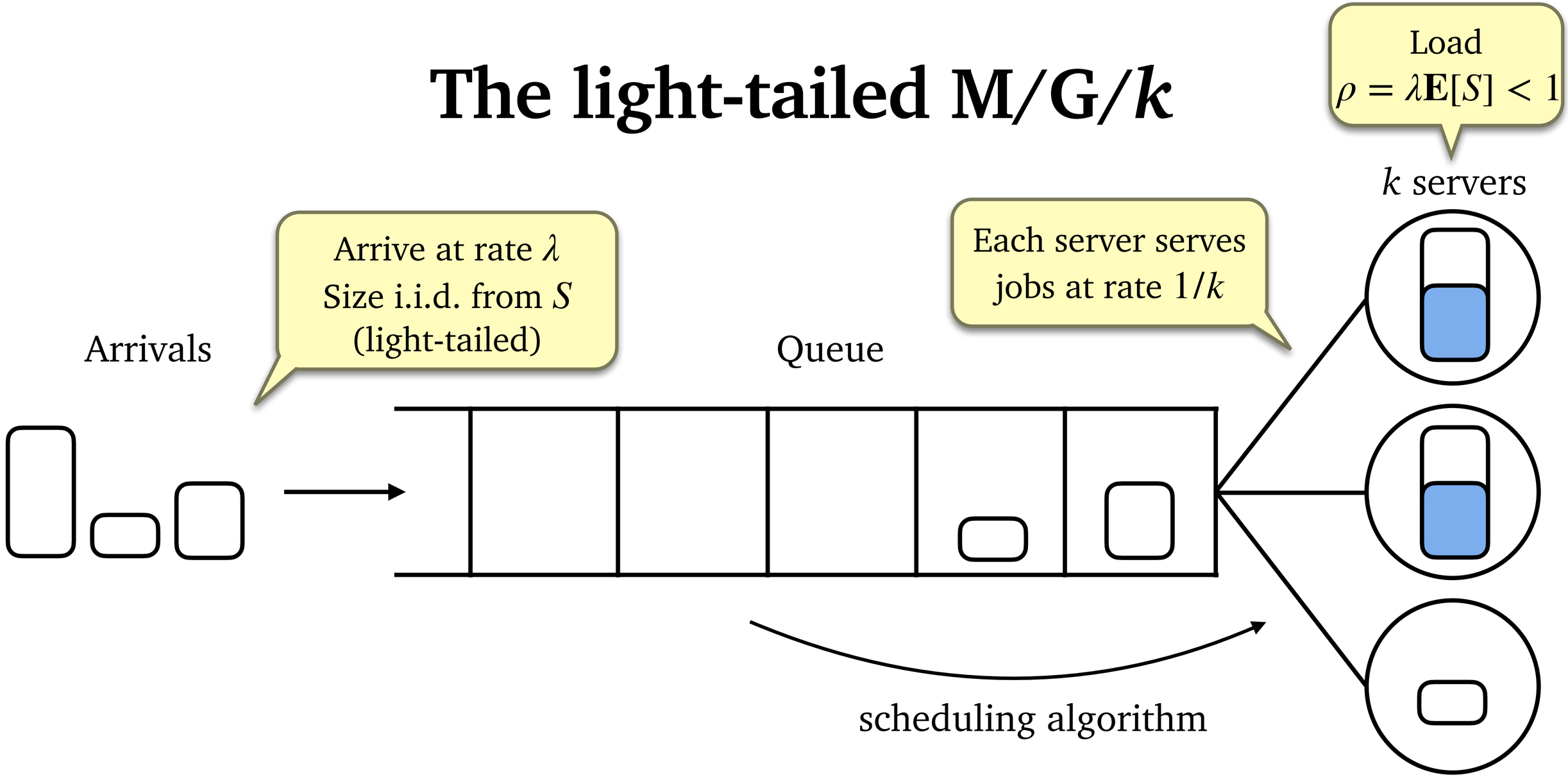

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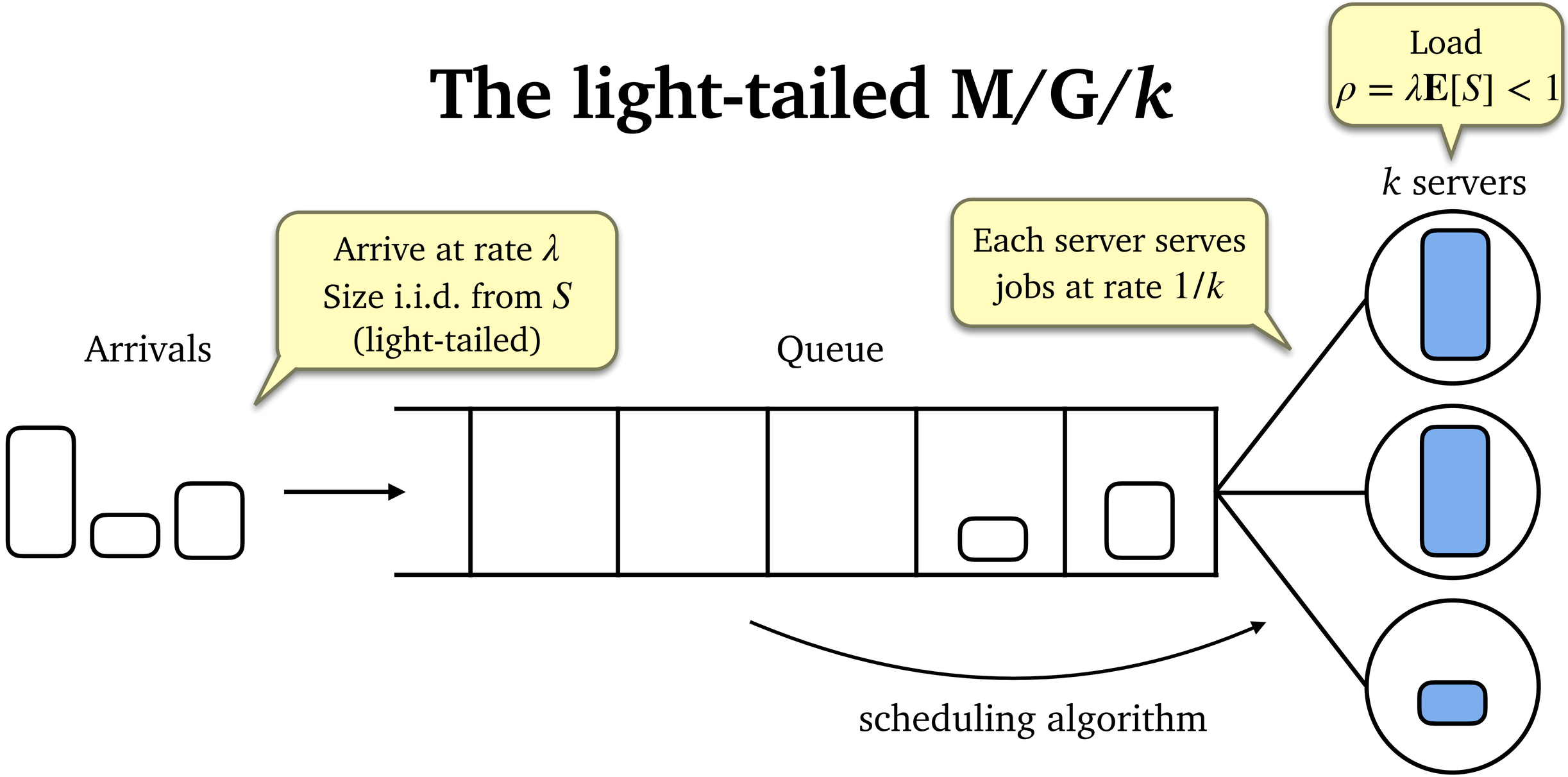

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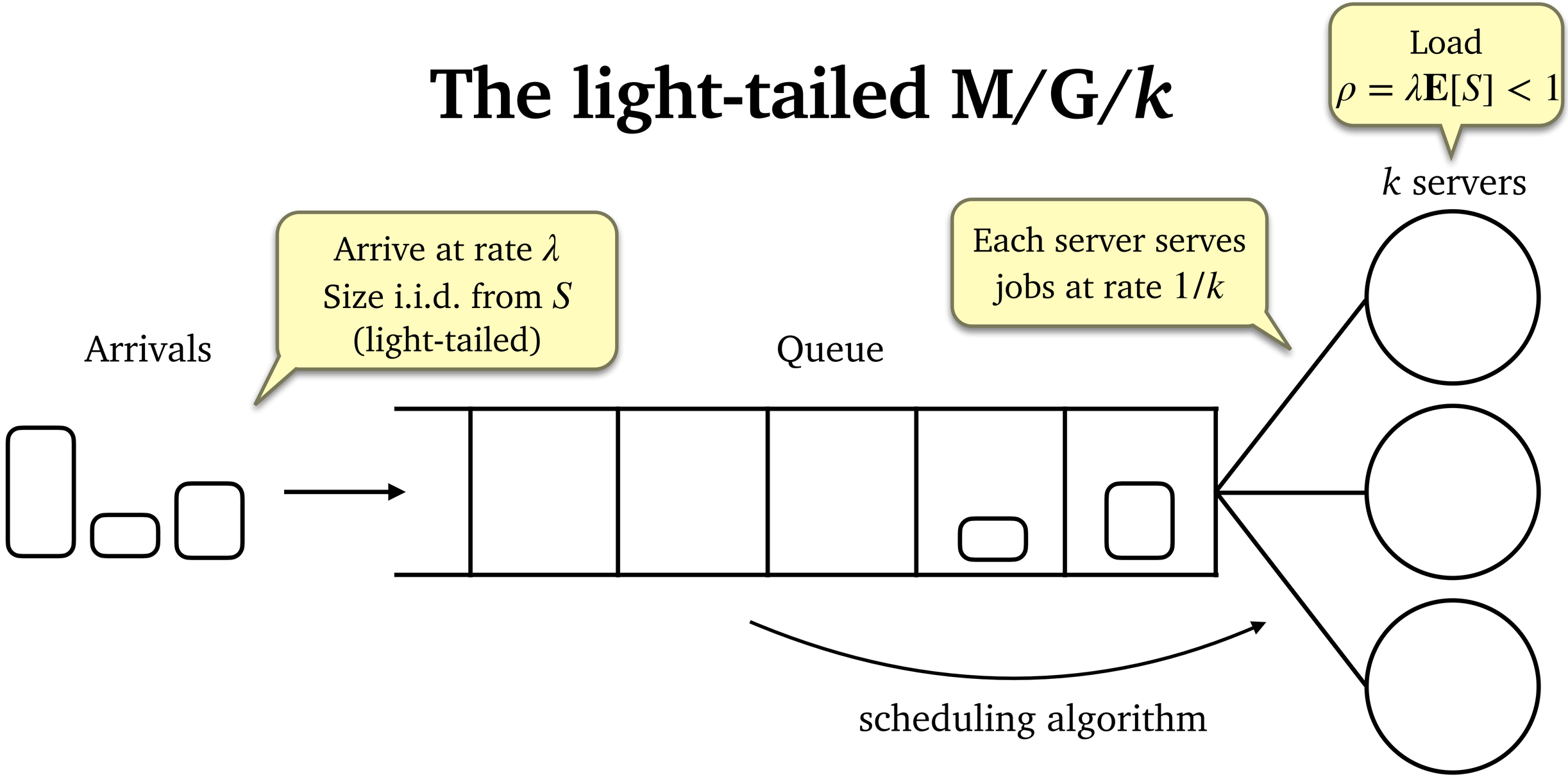

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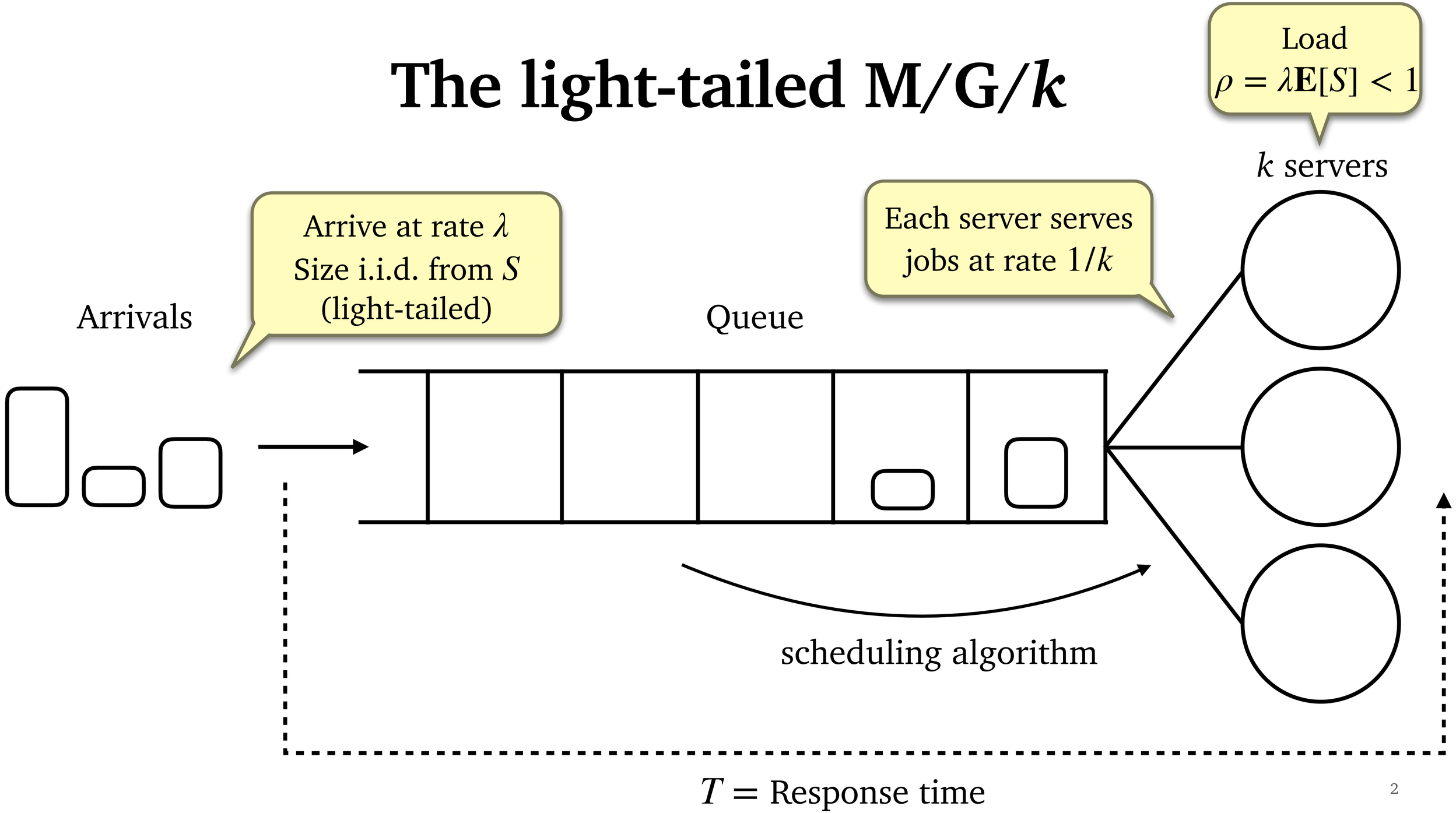

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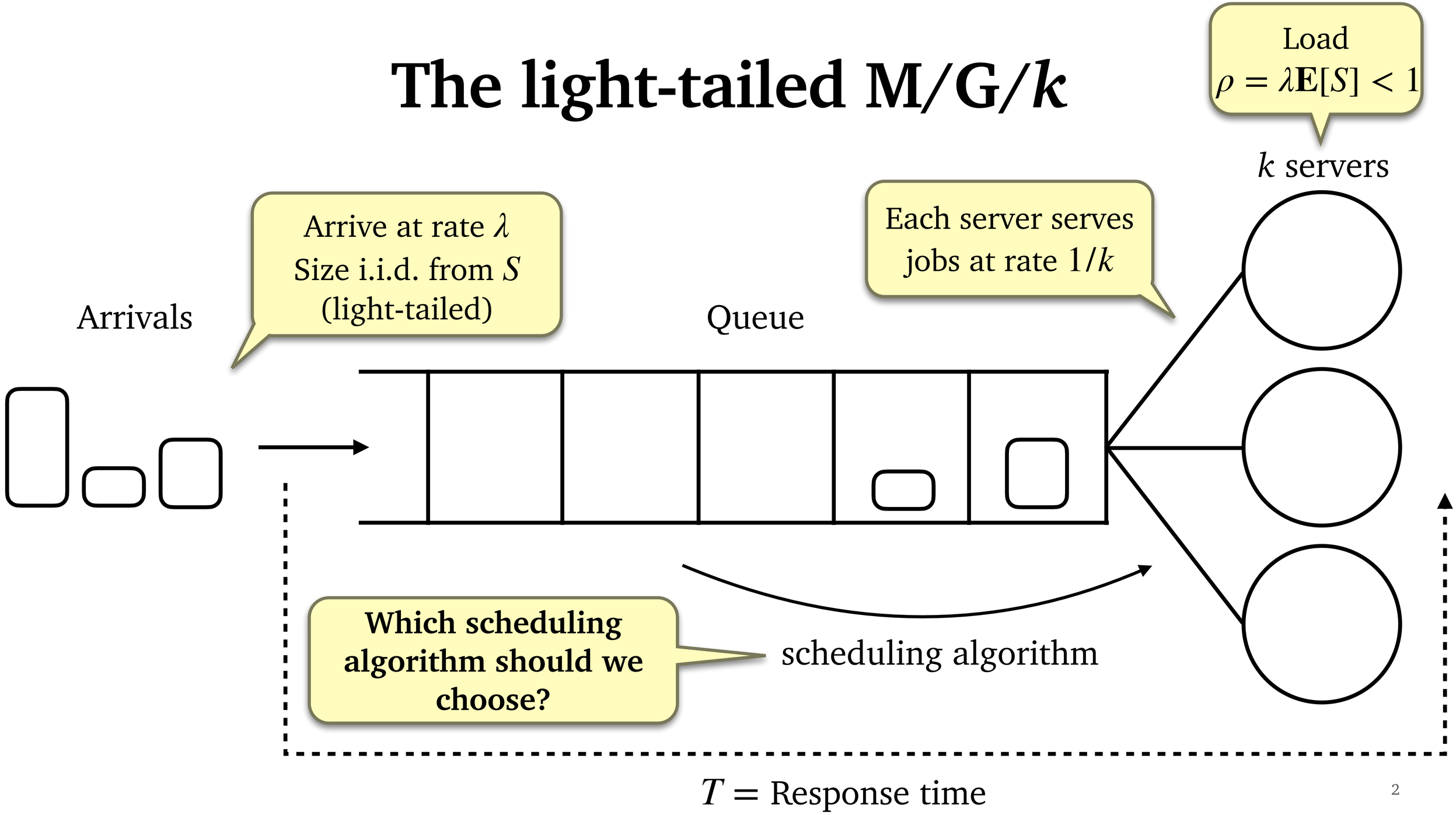

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In practice

[Yu & Scully 2024]

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[Grosof et al. 2018]

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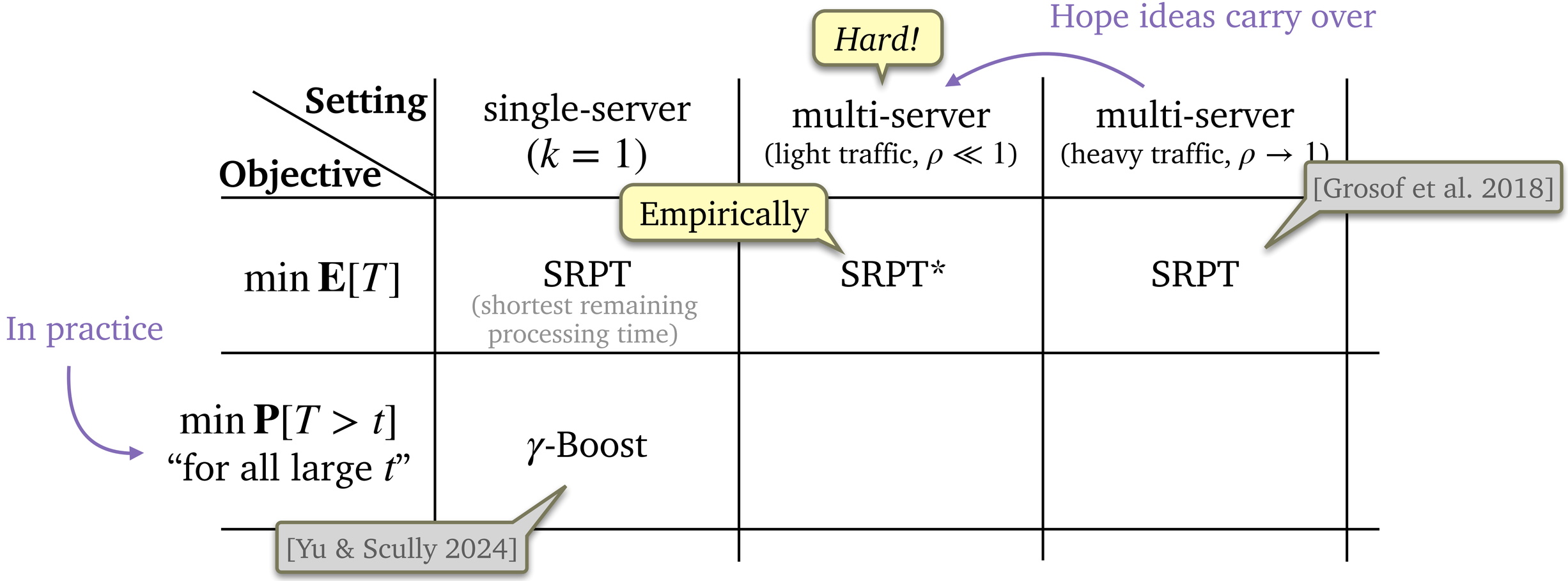

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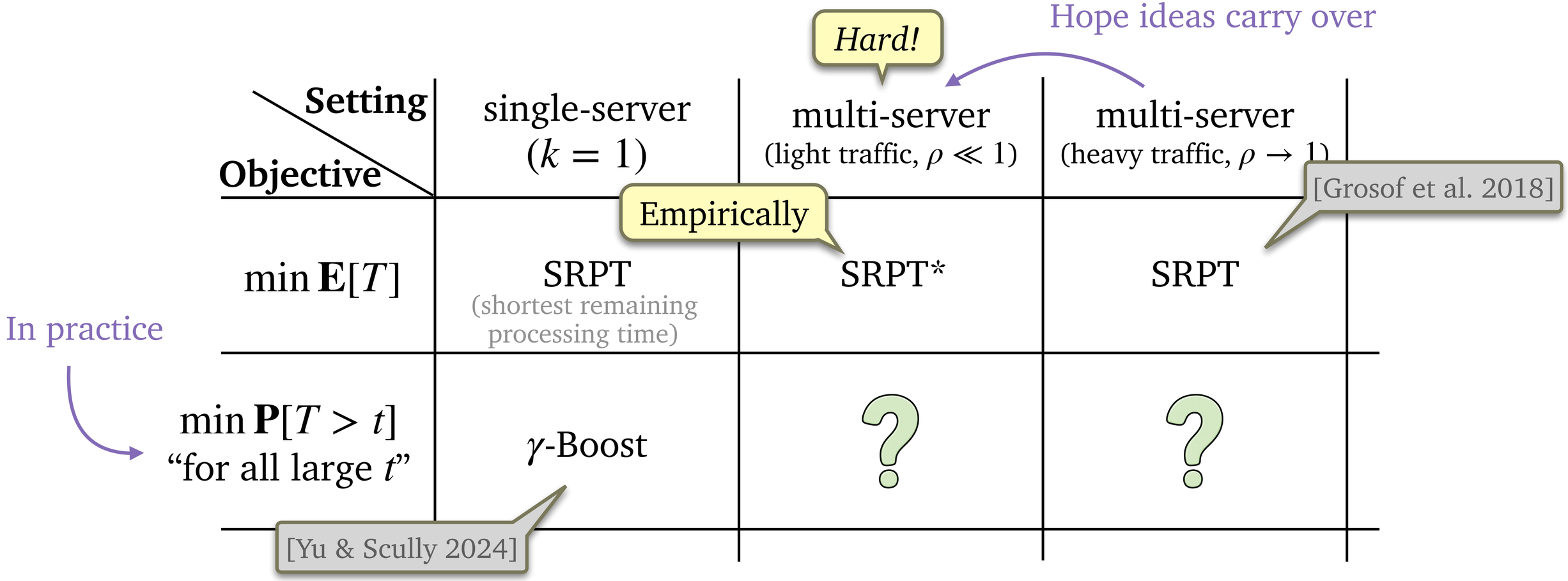

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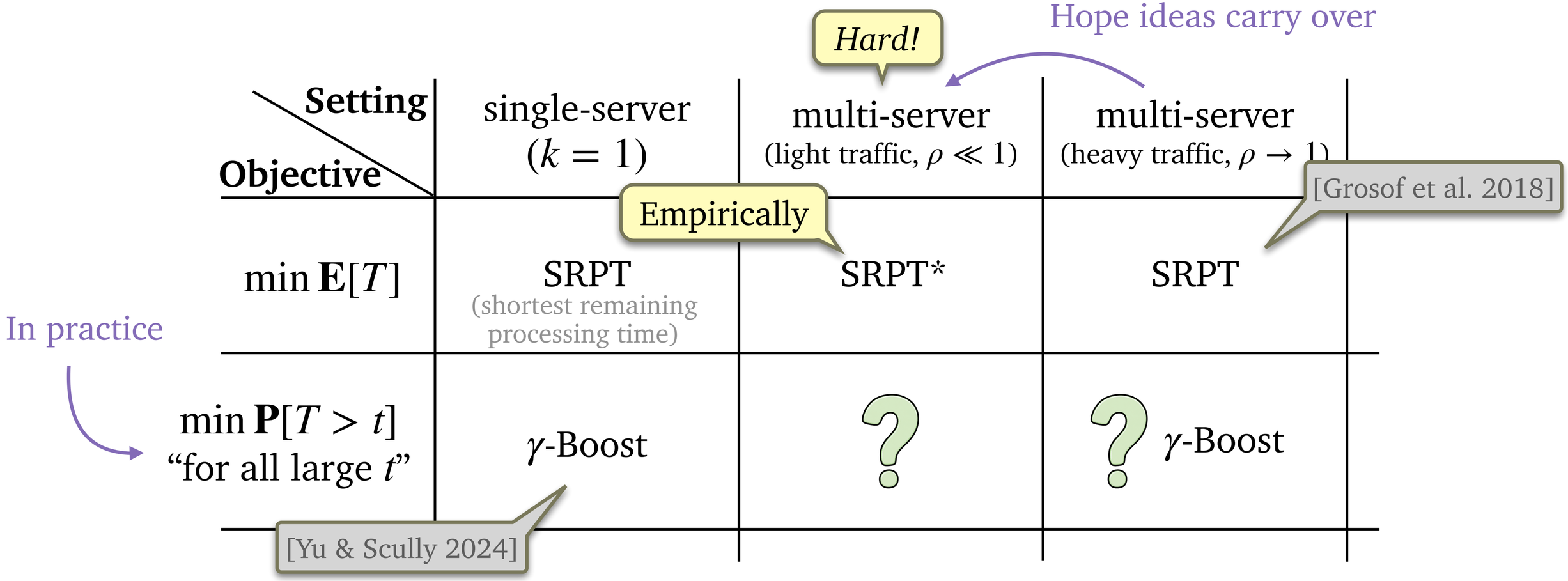

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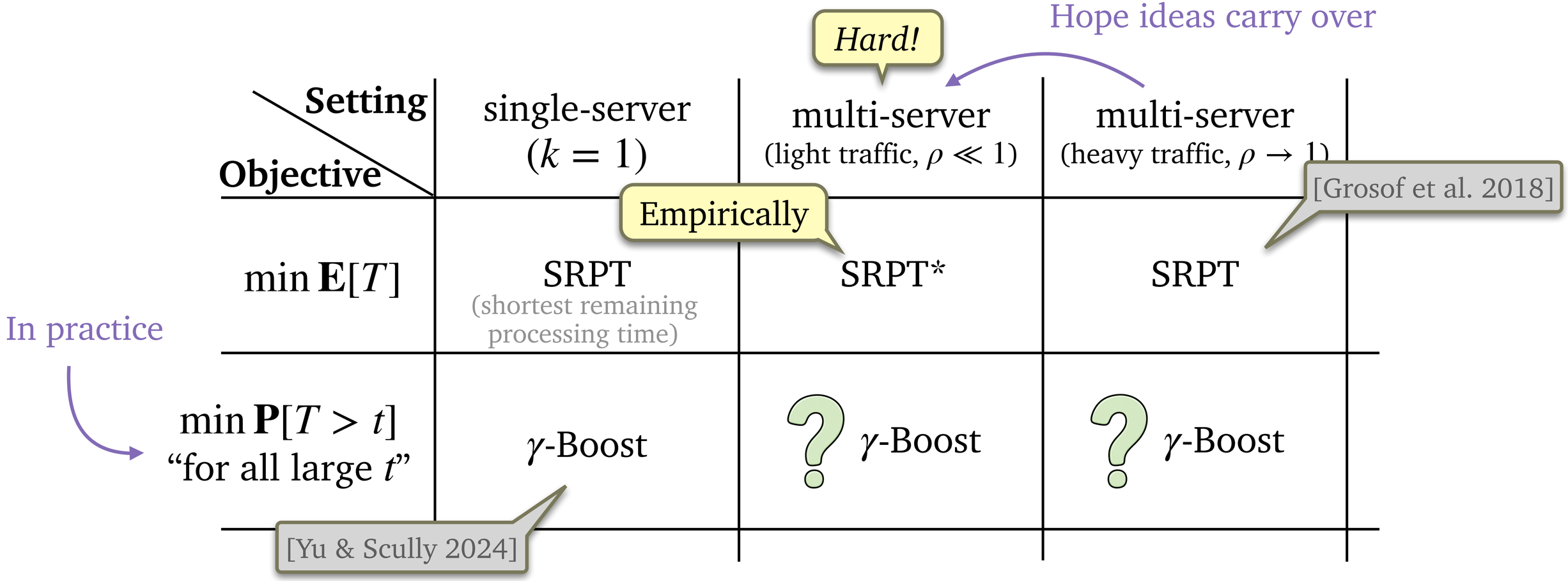

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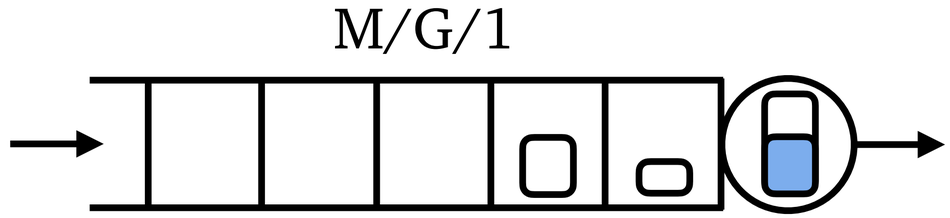

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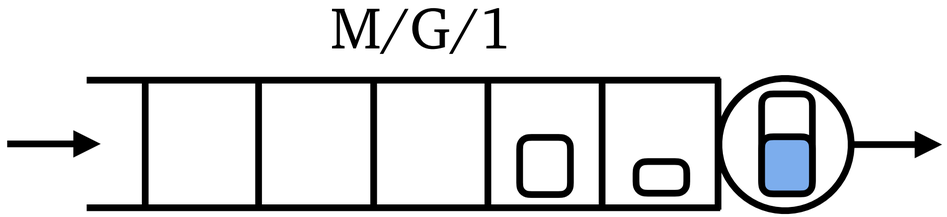

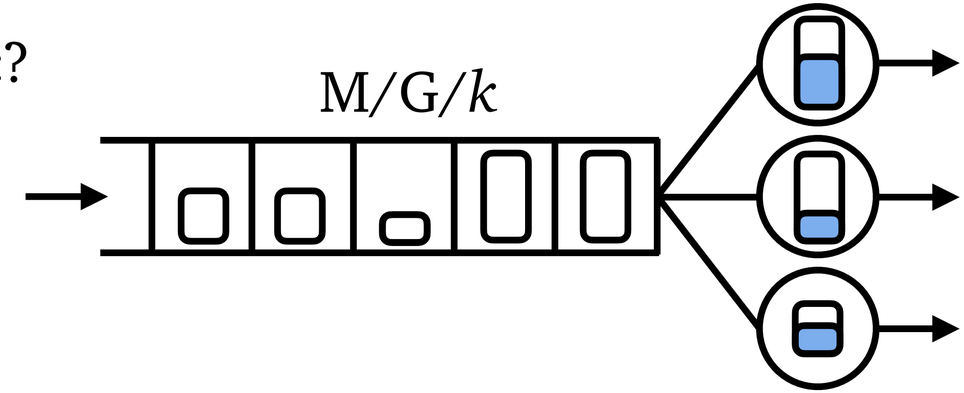

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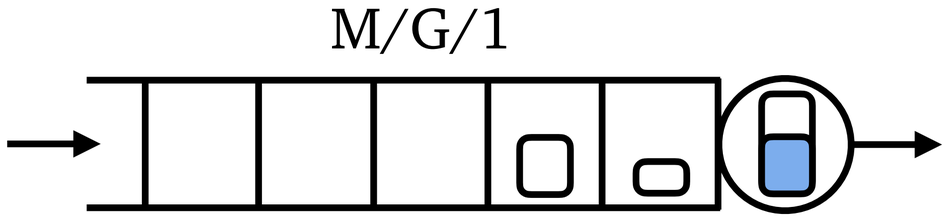

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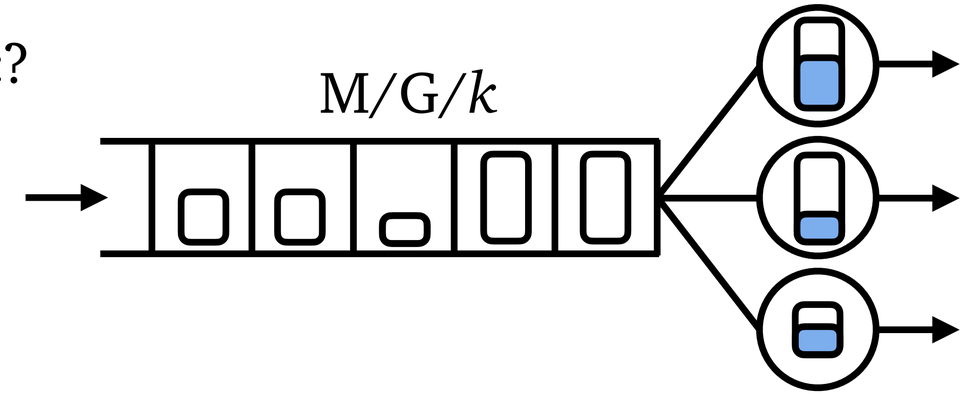

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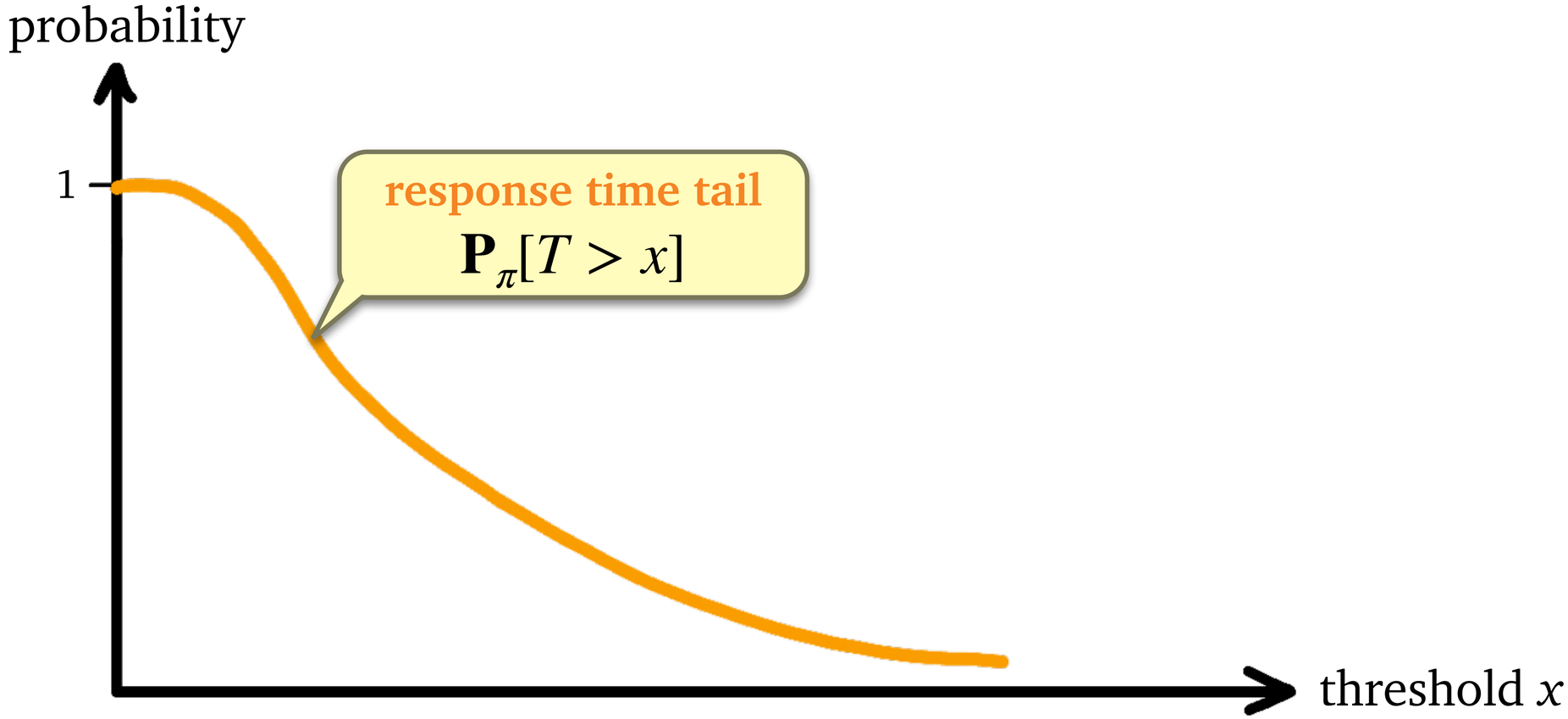

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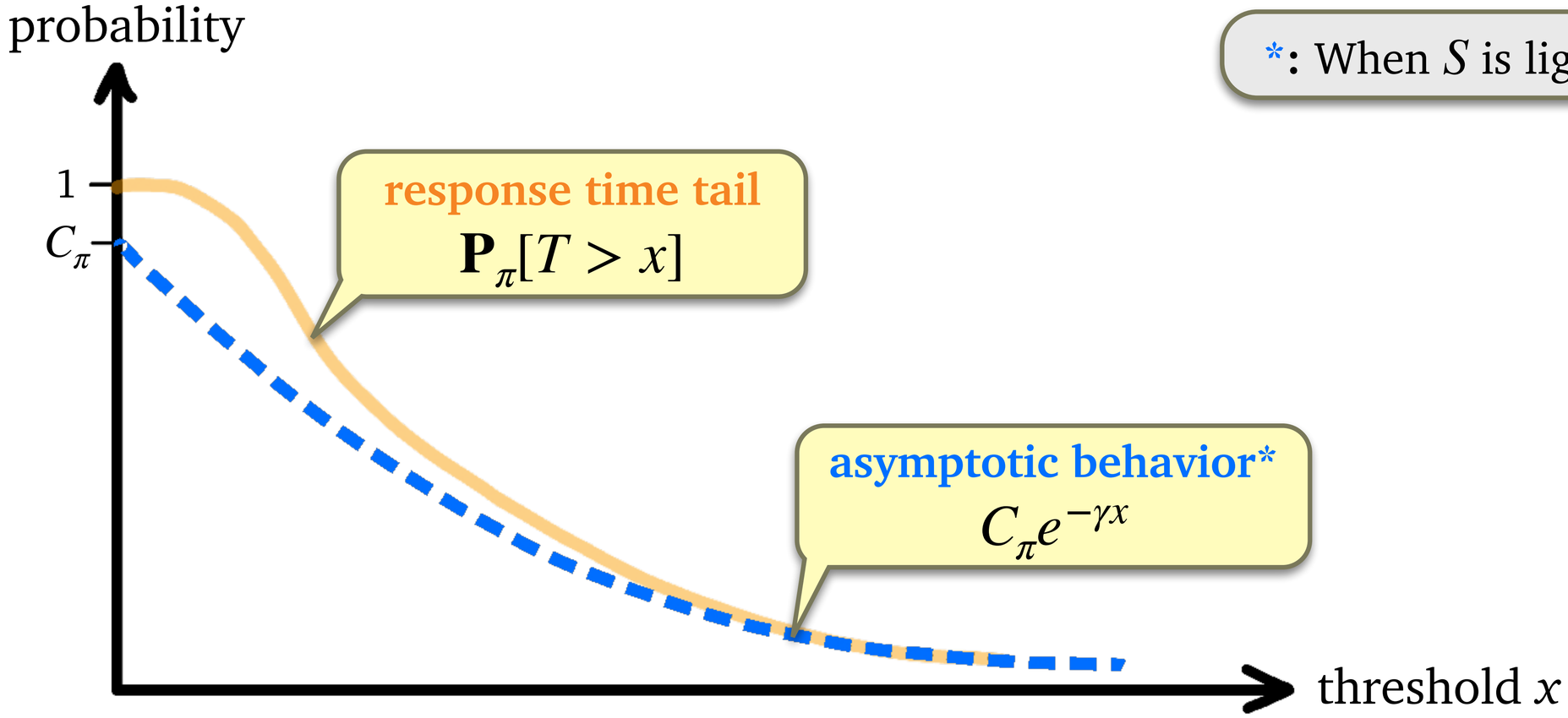

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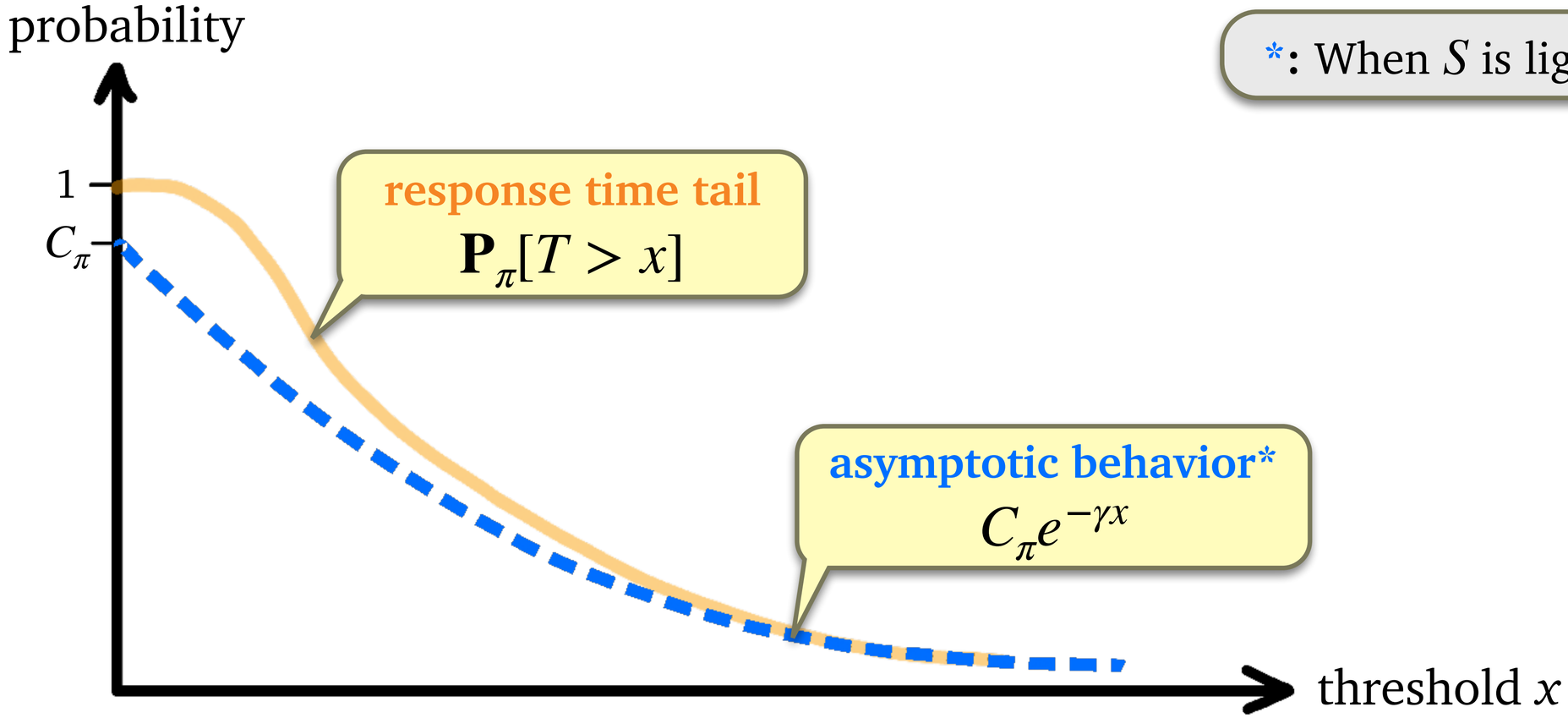

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Goal: find π that minimizes C_π

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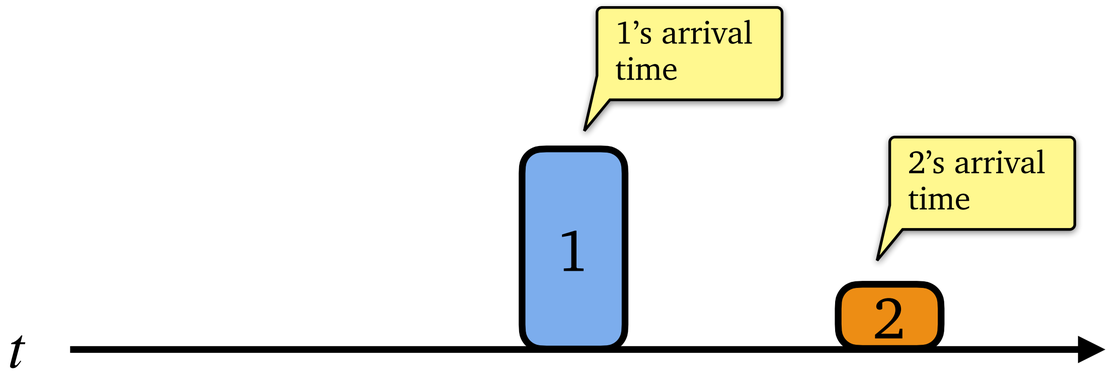

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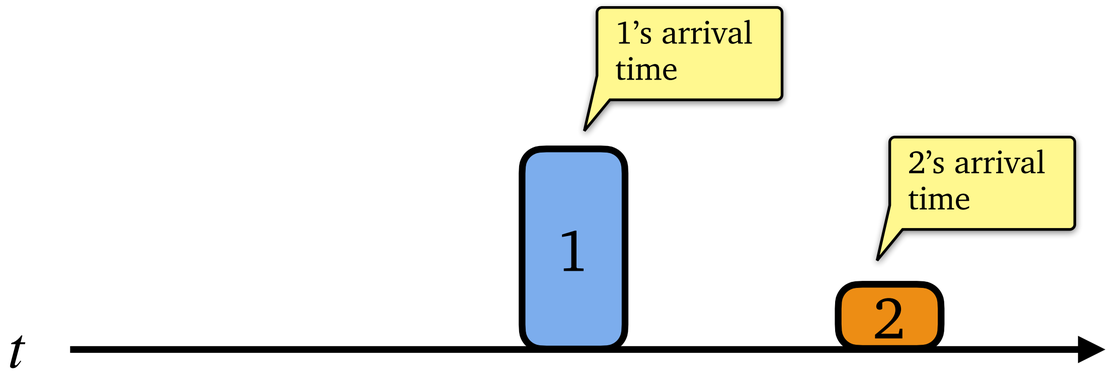

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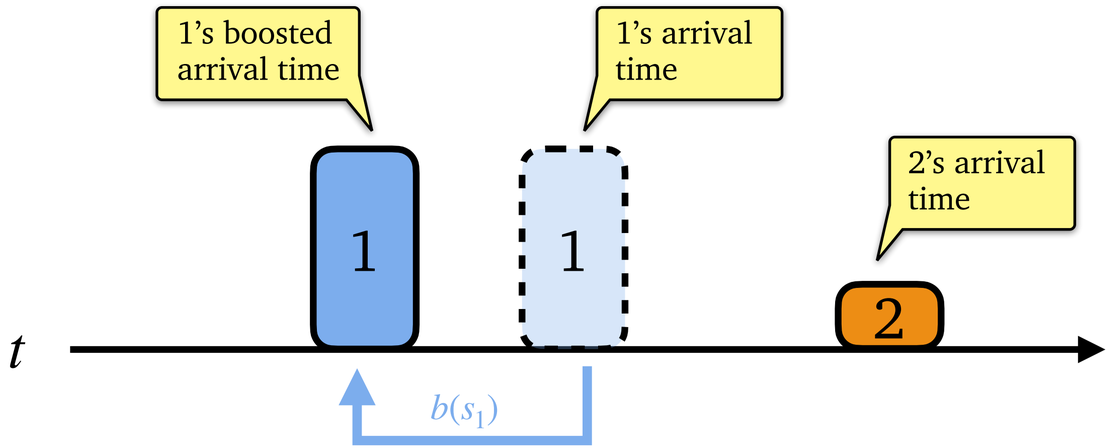

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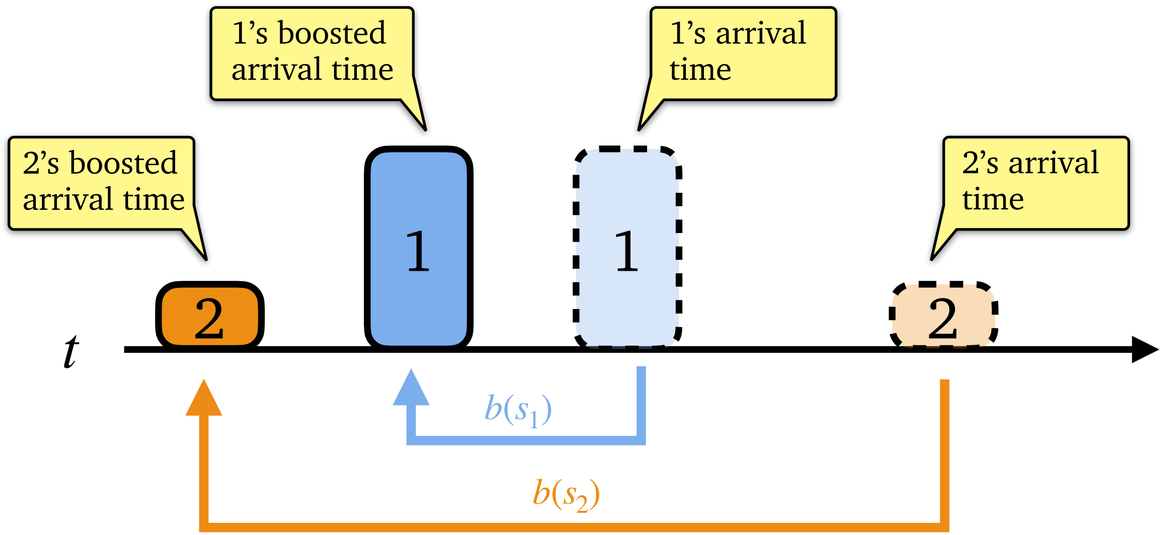

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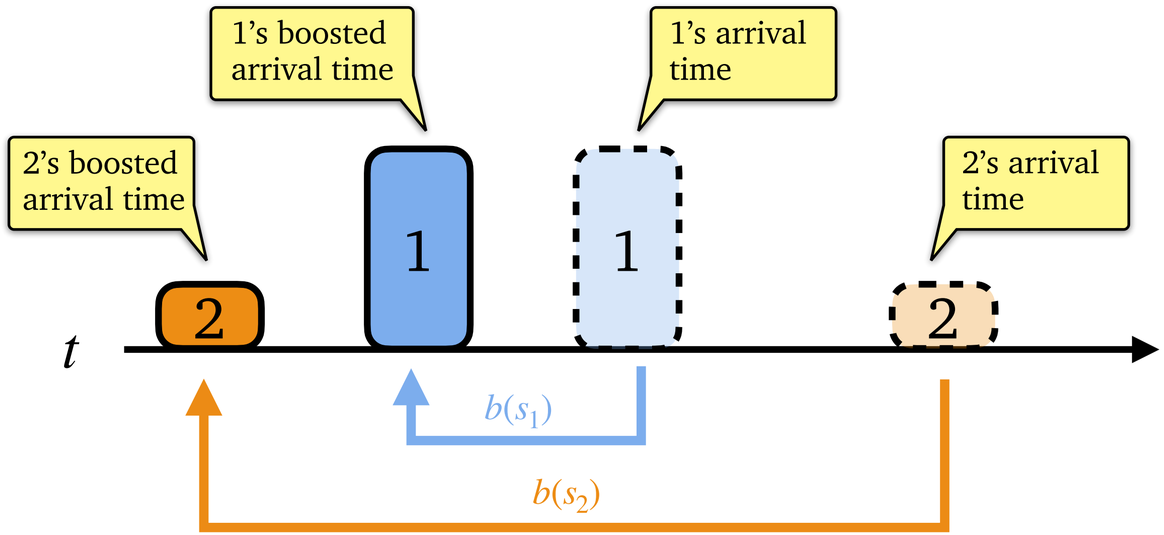

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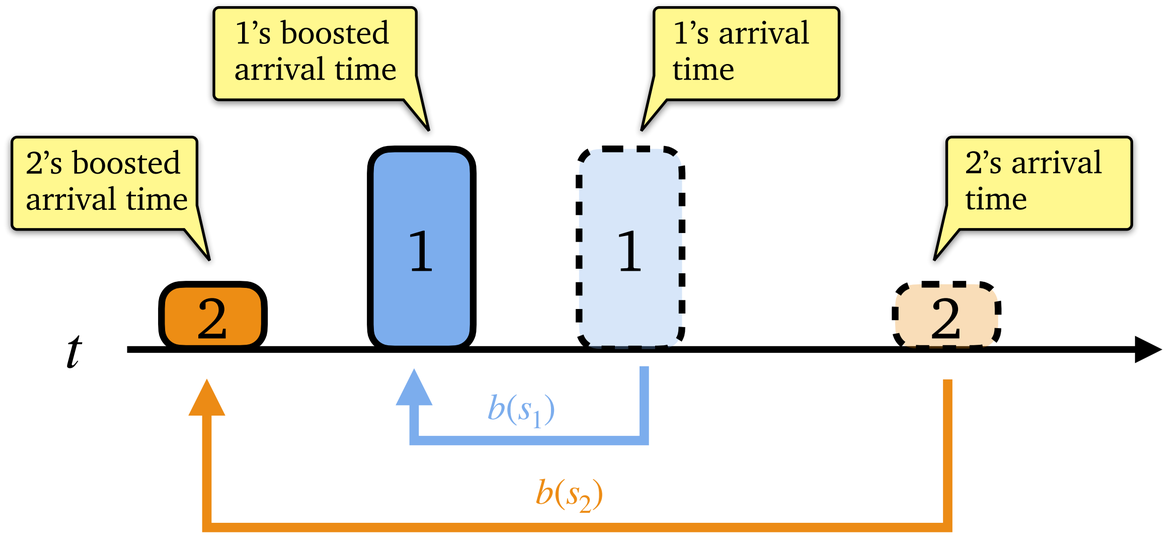

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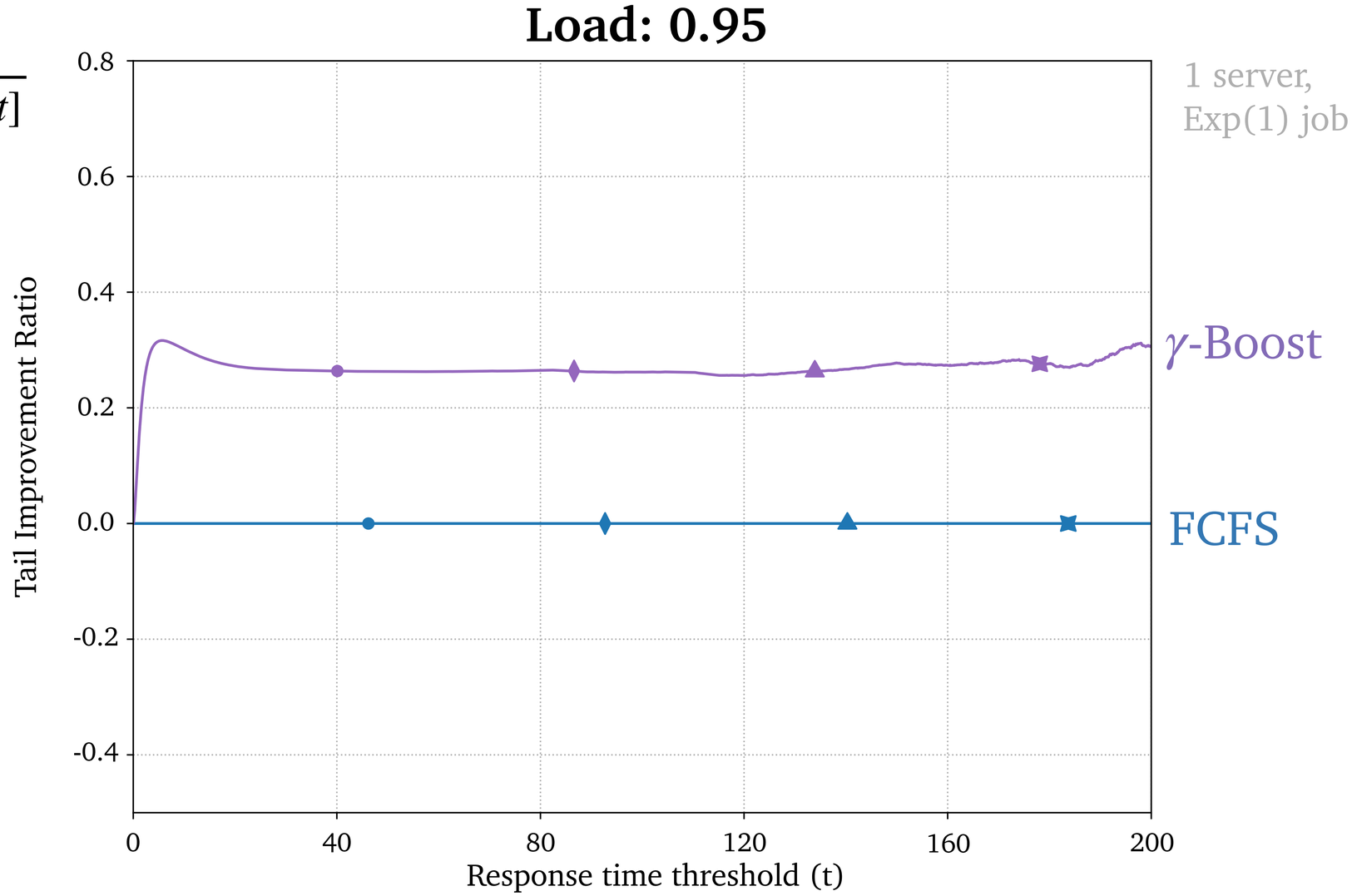
Theorem: in the M/G/1, $C_{\gamma\text{–Boost}} = \inf_{\pi} C_{\pi}$ [Yu & Scully 2024]

γ-Boost's performance in the M/G/1

$$1 - \frac{\mathbf{P}[T_\pi > t]}{\mathbf{P}[T_{\text{FCFS}} > t]}$$

(Higher is better)



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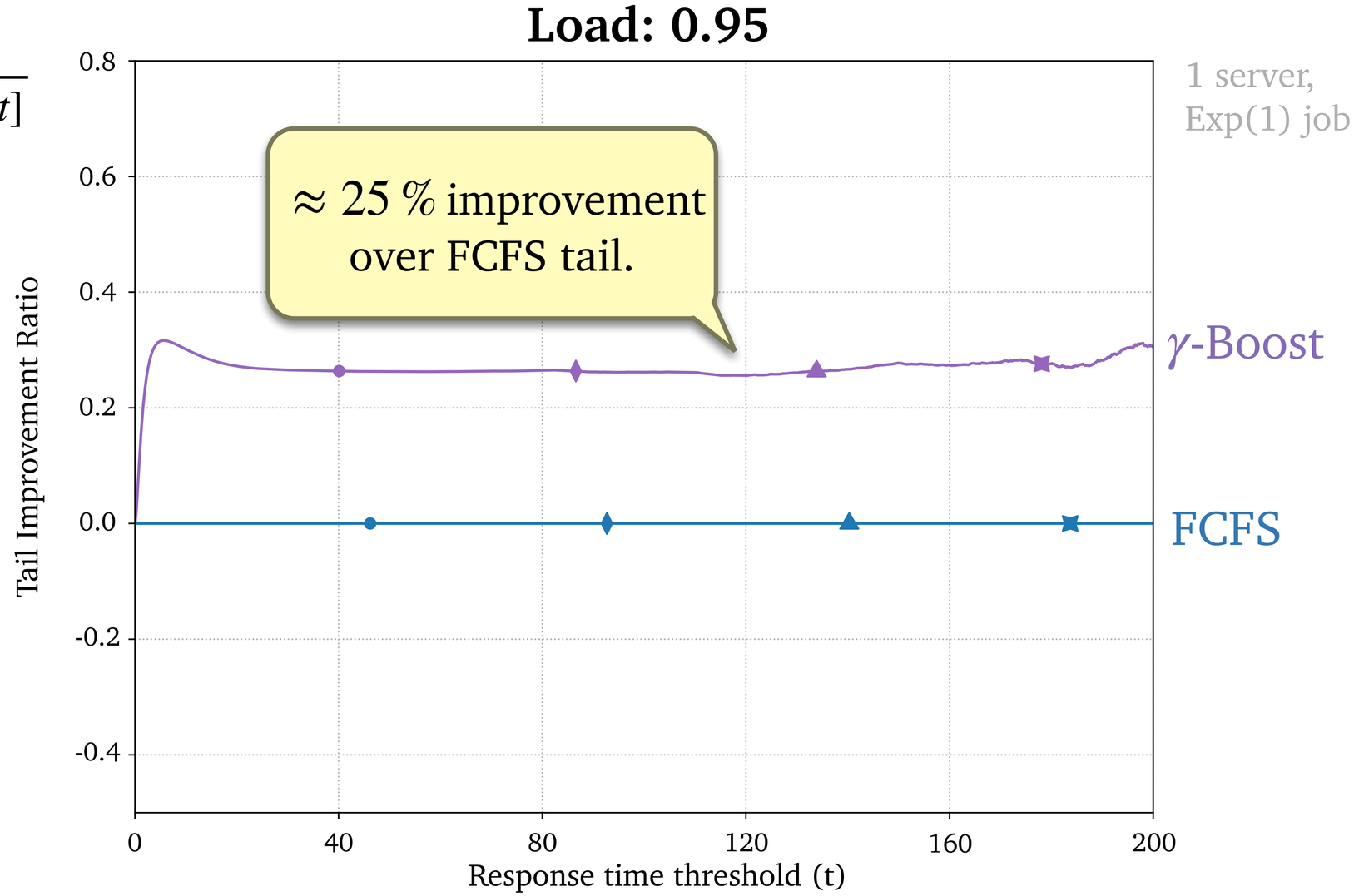

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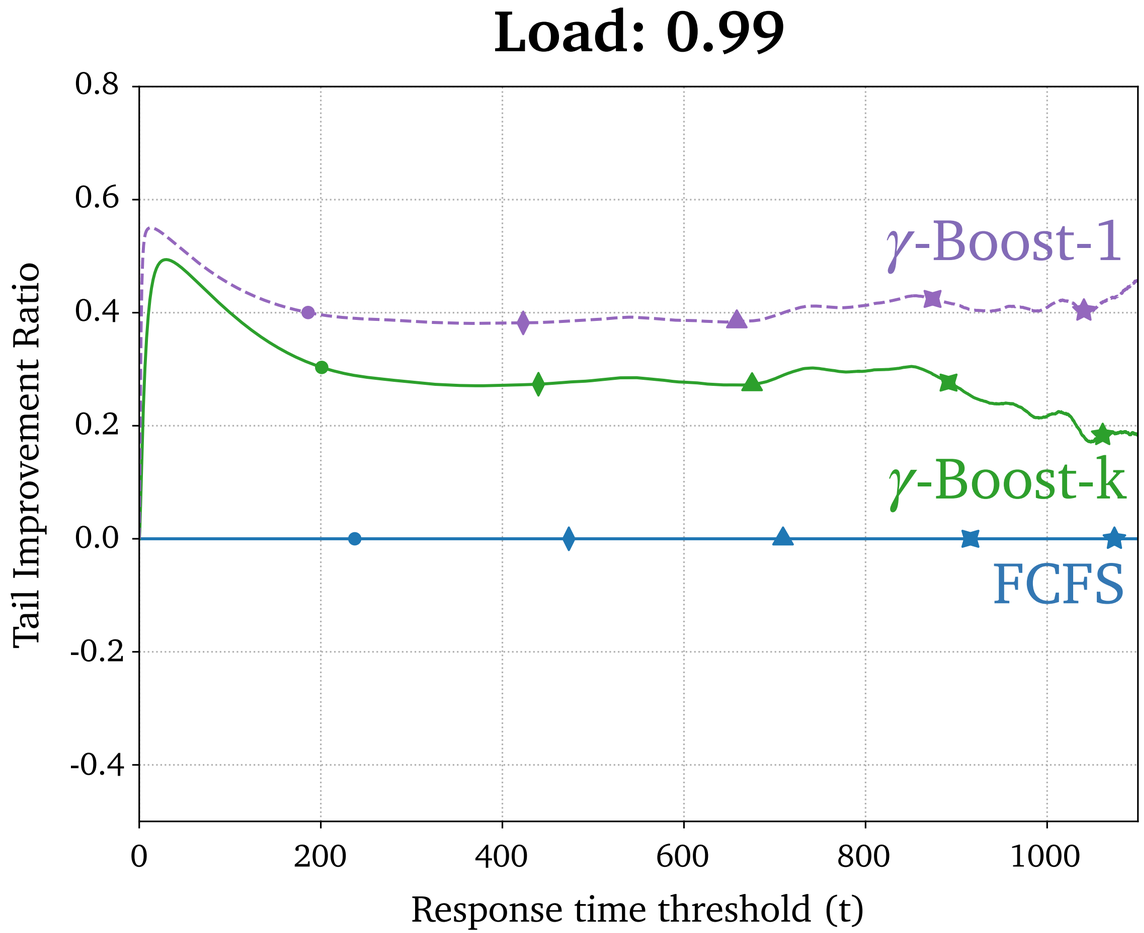

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Exp(1) job sizes

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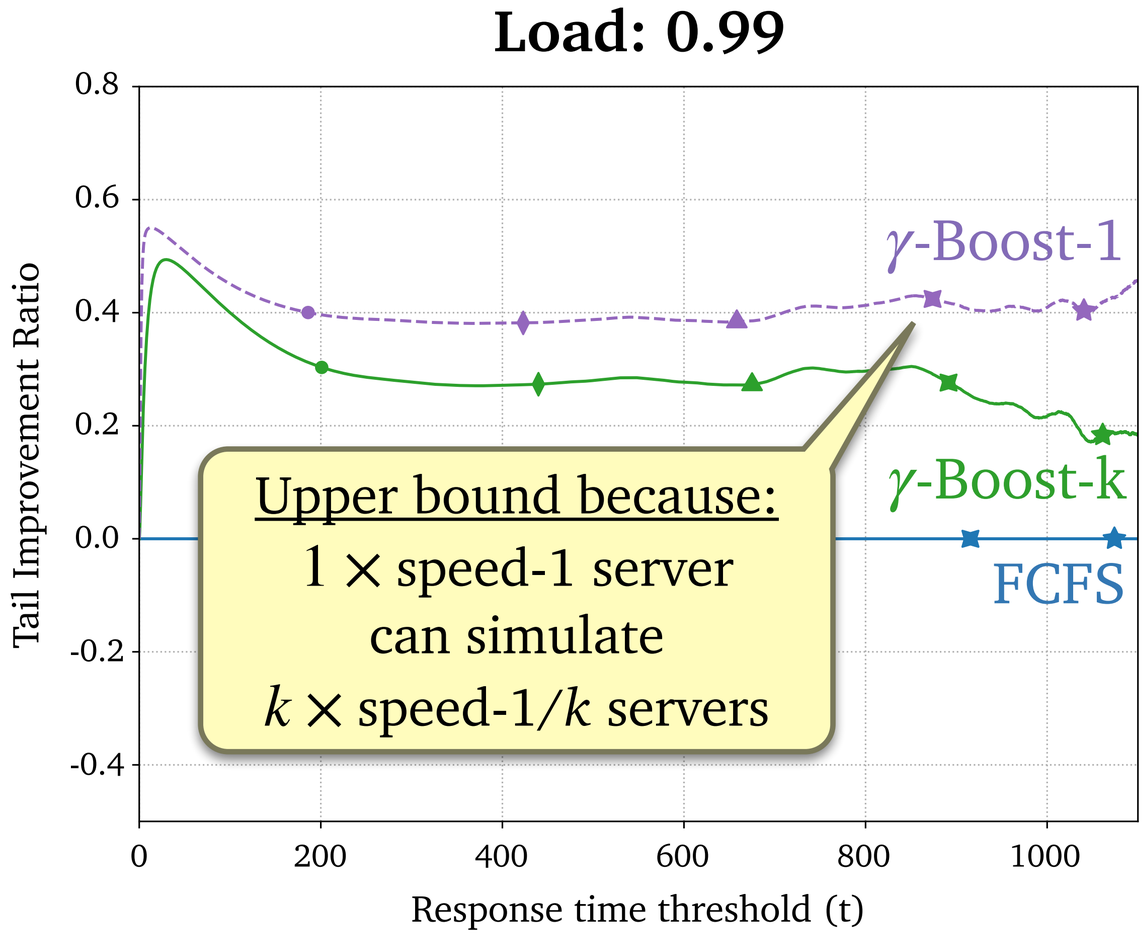

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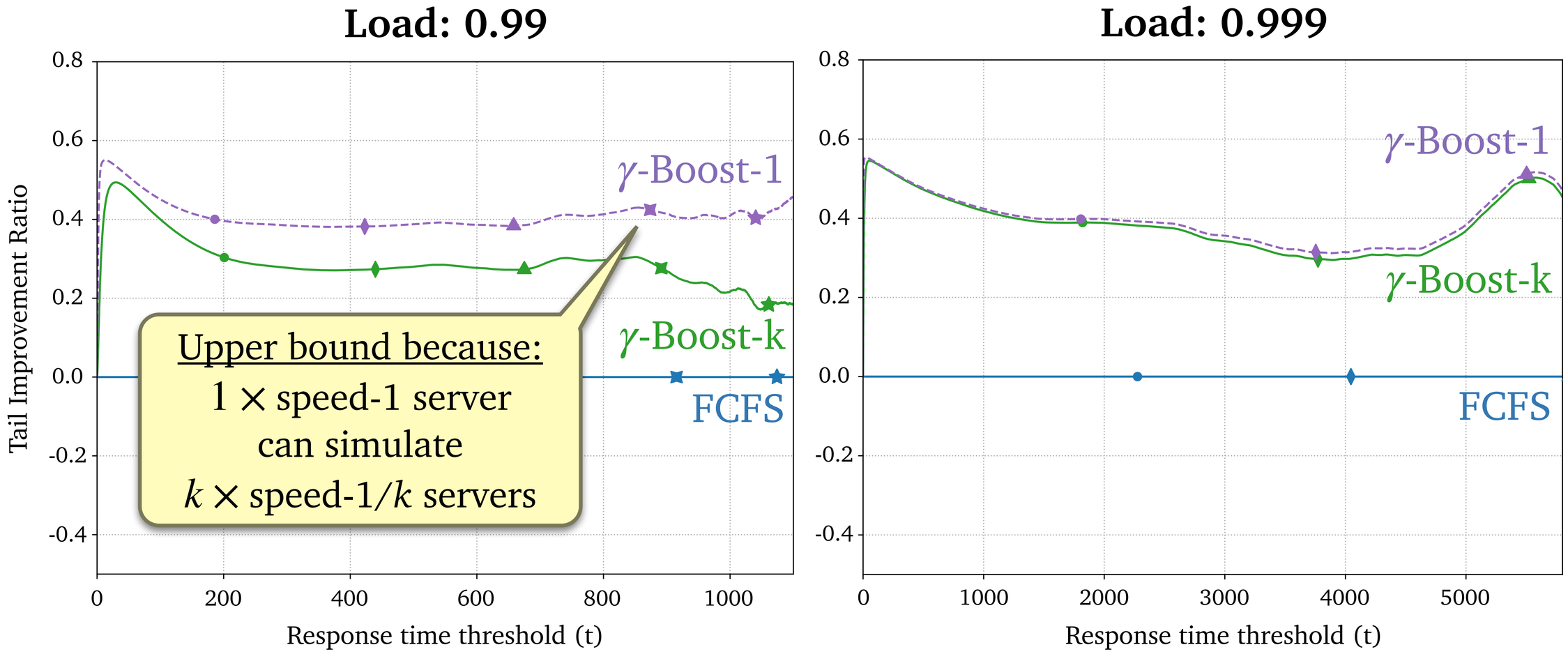

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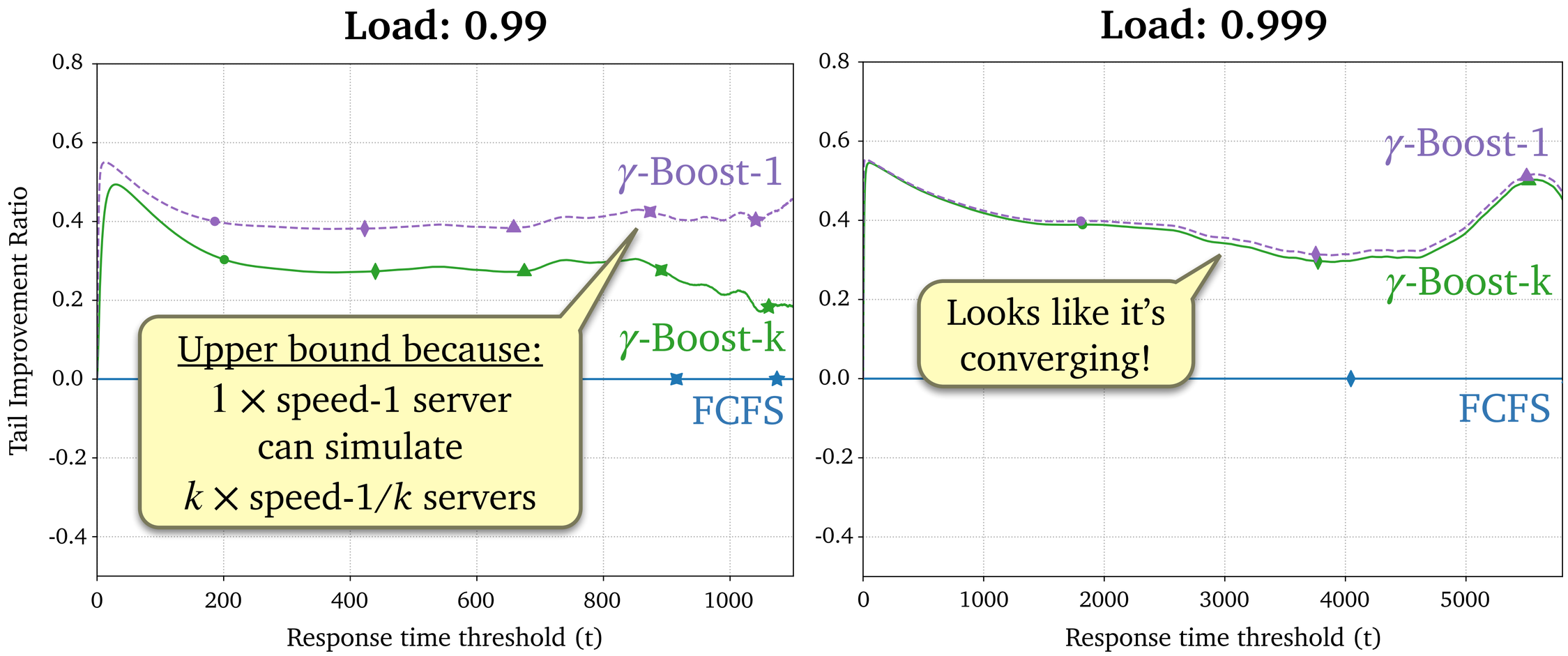

10 servers,
Exp(1) job sizes

Theorem: $\lim_{\rho \to 1} C_{\gamma\text{-Boost-k}} = C_{\gamma\text{-Boost-1}}$

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For all work-conserving policies!

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George's Lemma

For all work-conserving policies!

This talk

What do we mean by $\min \mathbf{P}[T > t]$ "for all large t"?

What is the γ-Boost policy?

Is γ-Boost tail optimal in the M/G/k in heavy traffic?

Does γ-Boost have good empirical tail performance in the M/G/k in lighter traffic?

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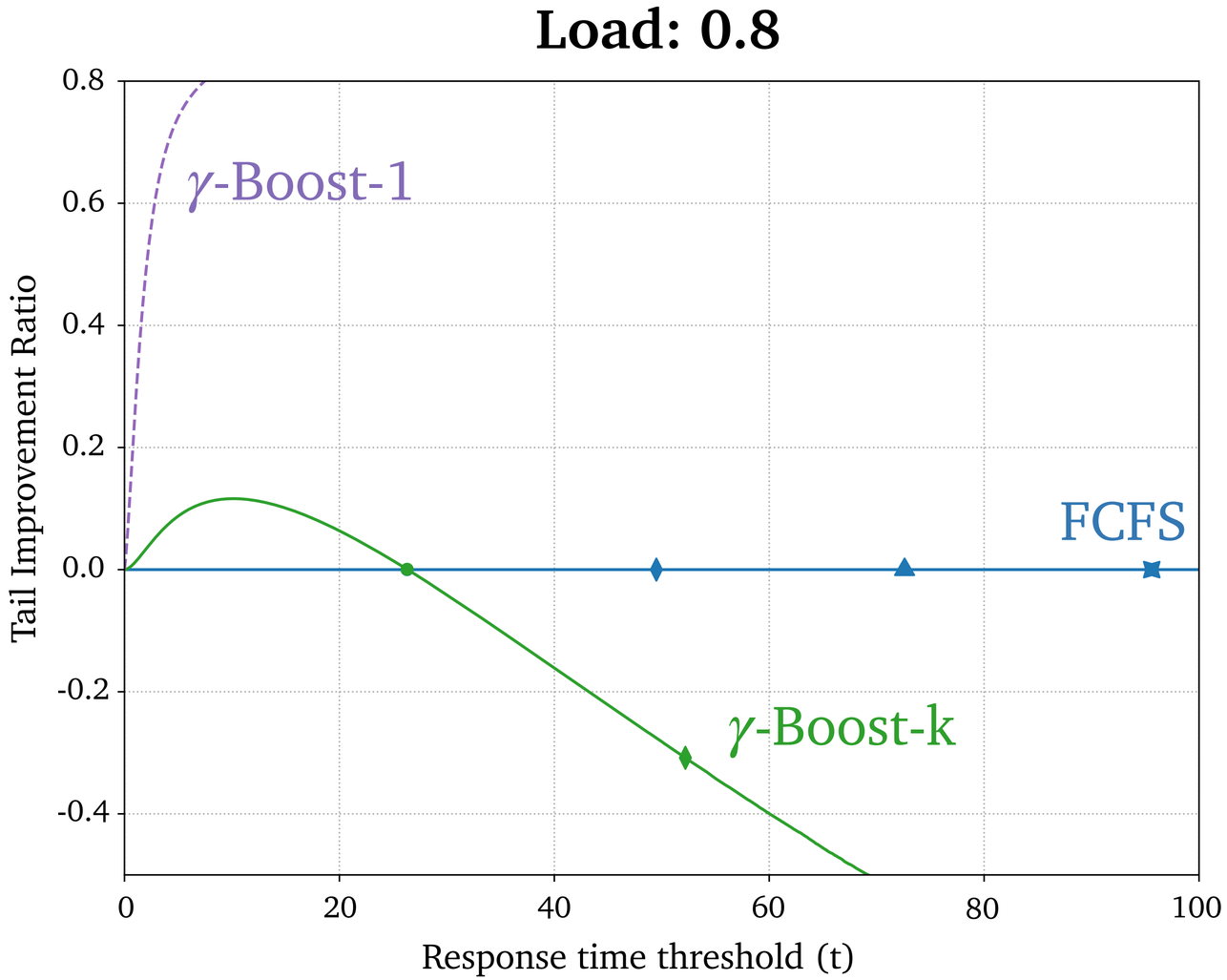
γ-Boost performance in lighter traffic



10 servers,
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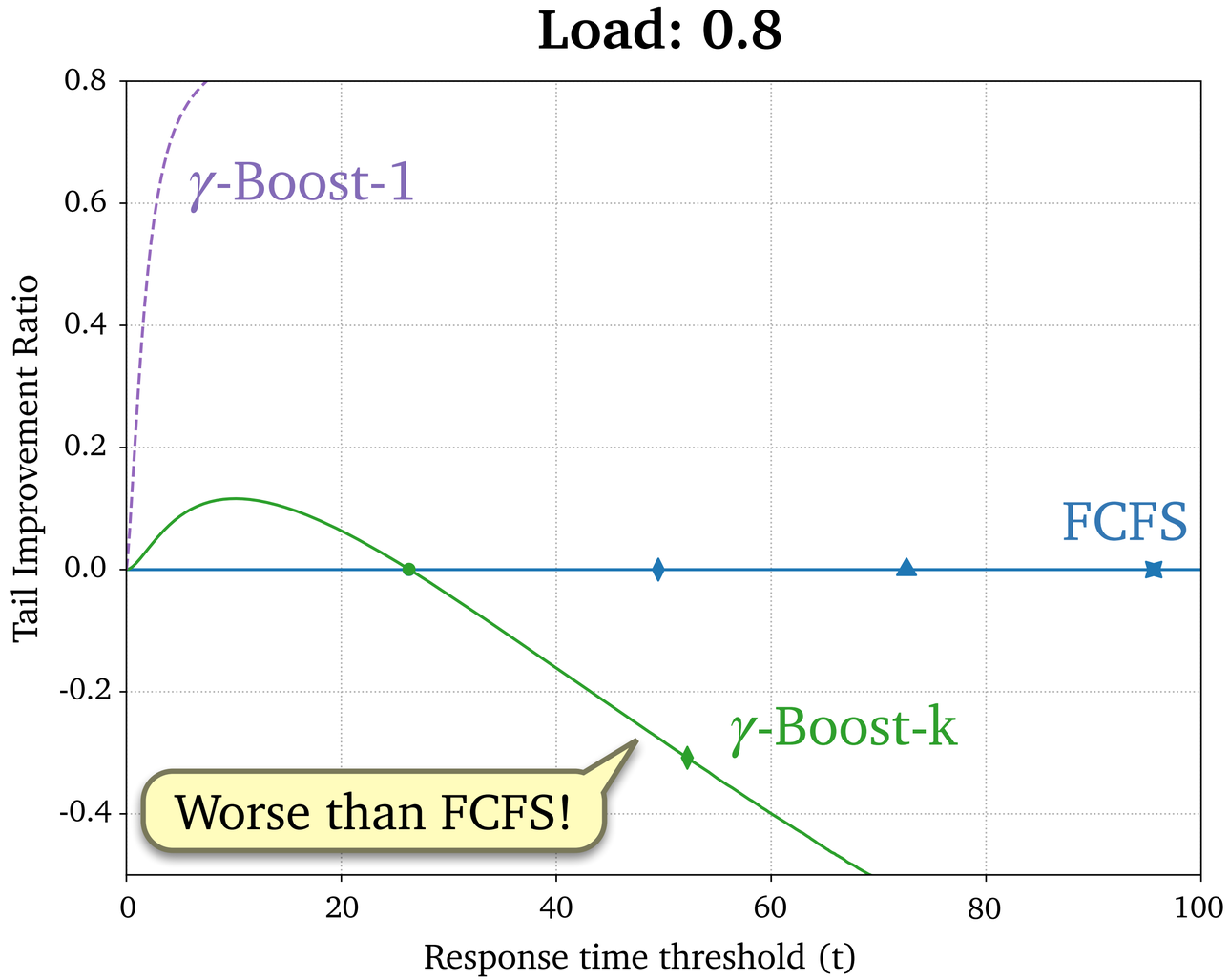
γ-Boost performance in lighter traffic



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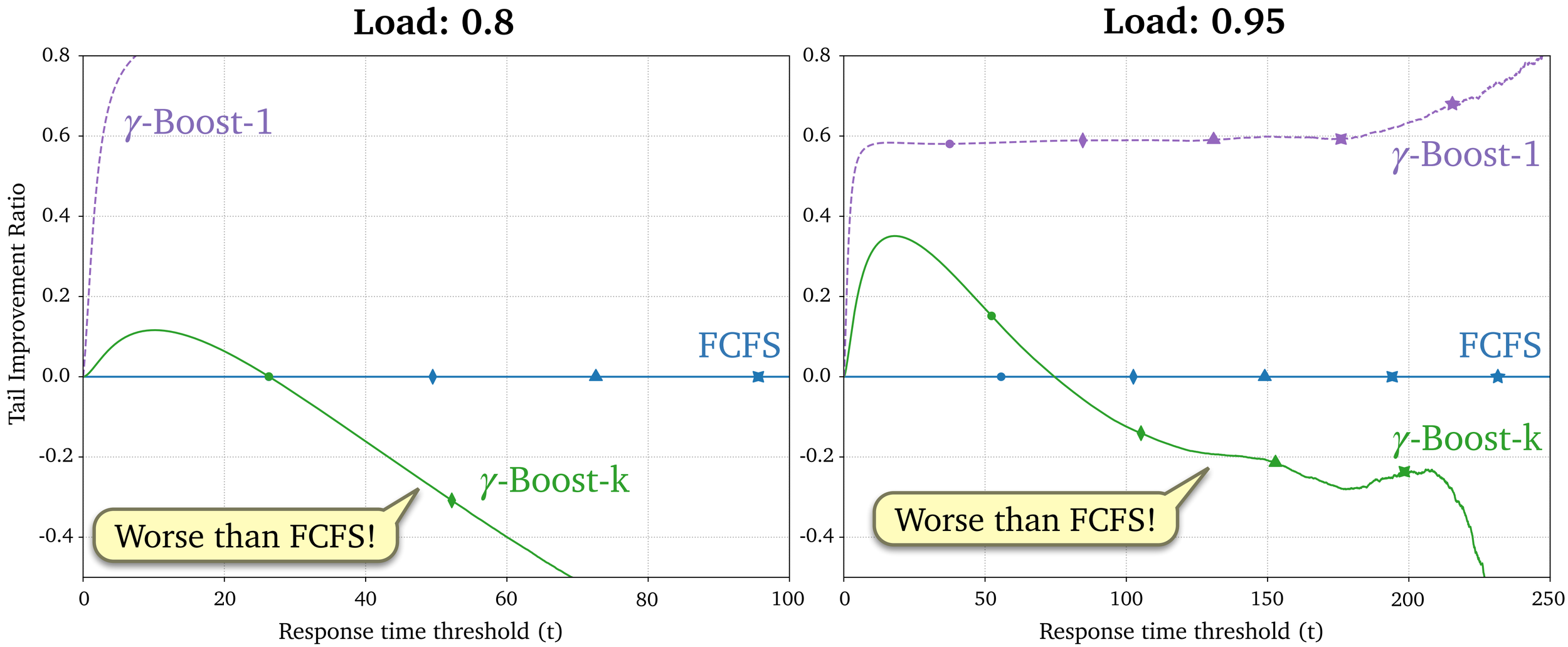

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Yes!

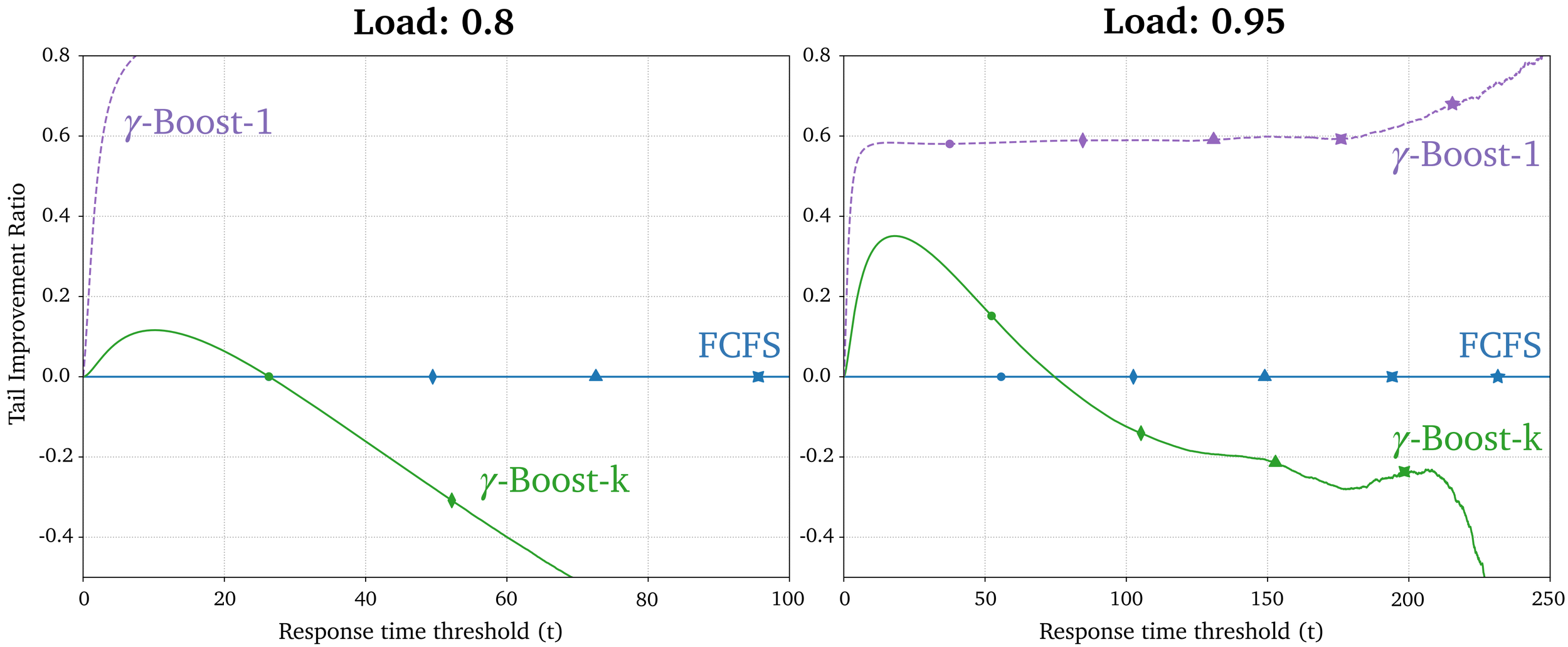
Does γ-Boost have [illegible] ance
in the M/G/k in lighter traffic?

What should we do instead?

No..

womp womp

γ-Boost performance in lighter traffic



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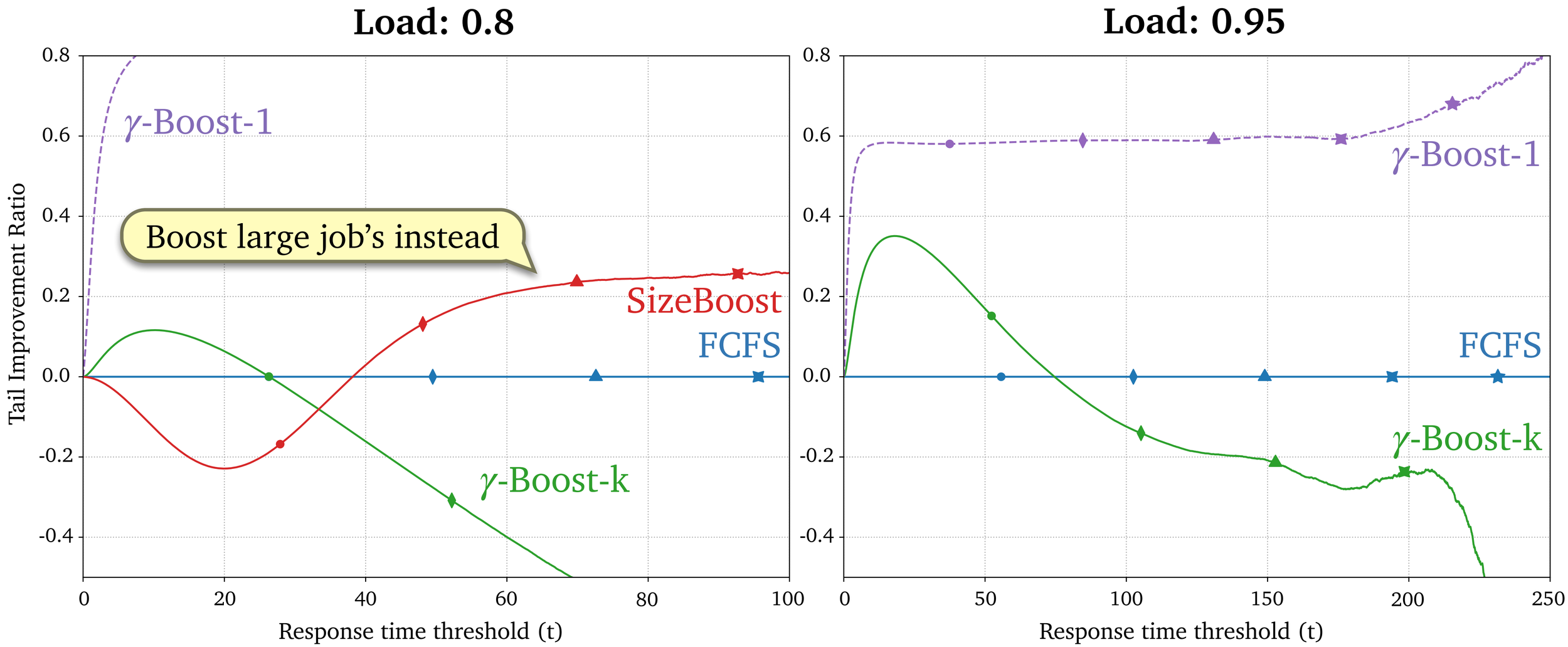

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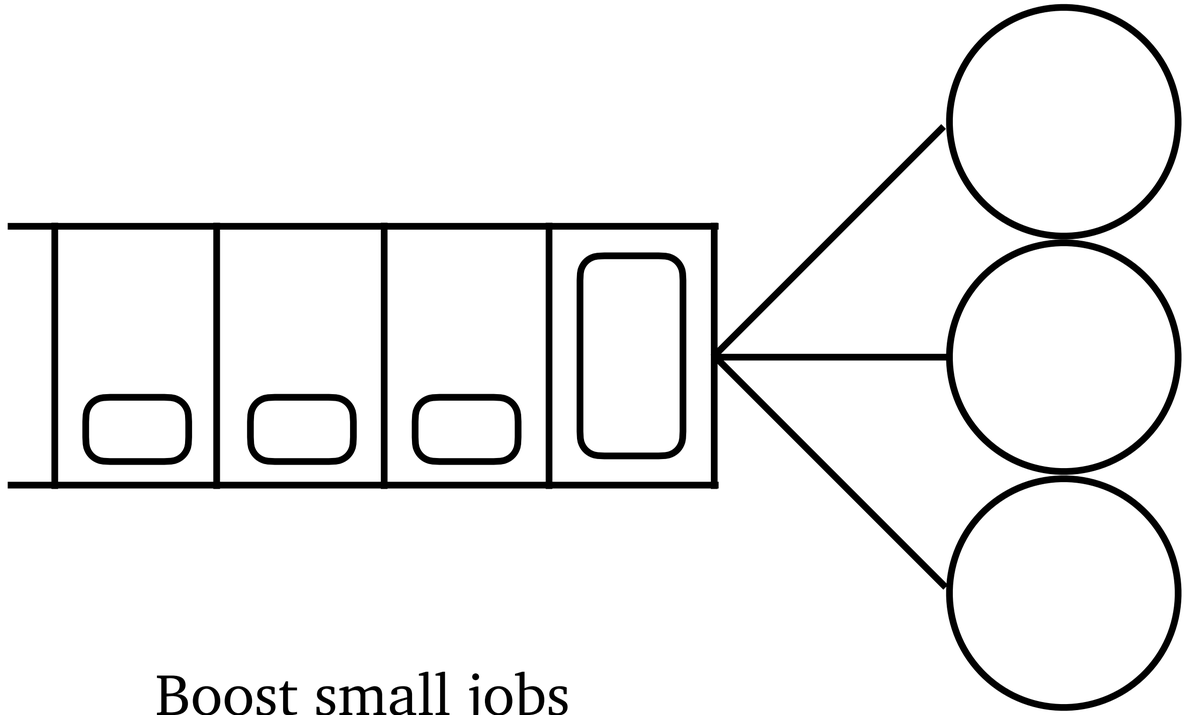
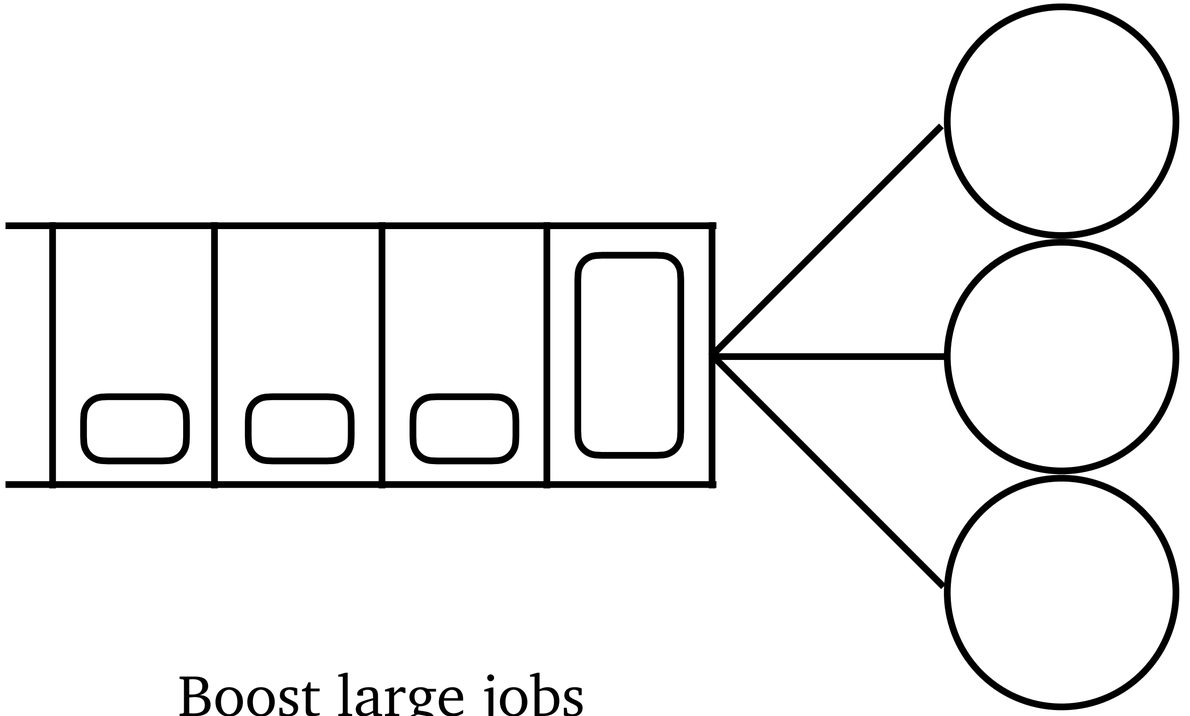
Why might SizeBoost make sense at low load?



Boost small jobs



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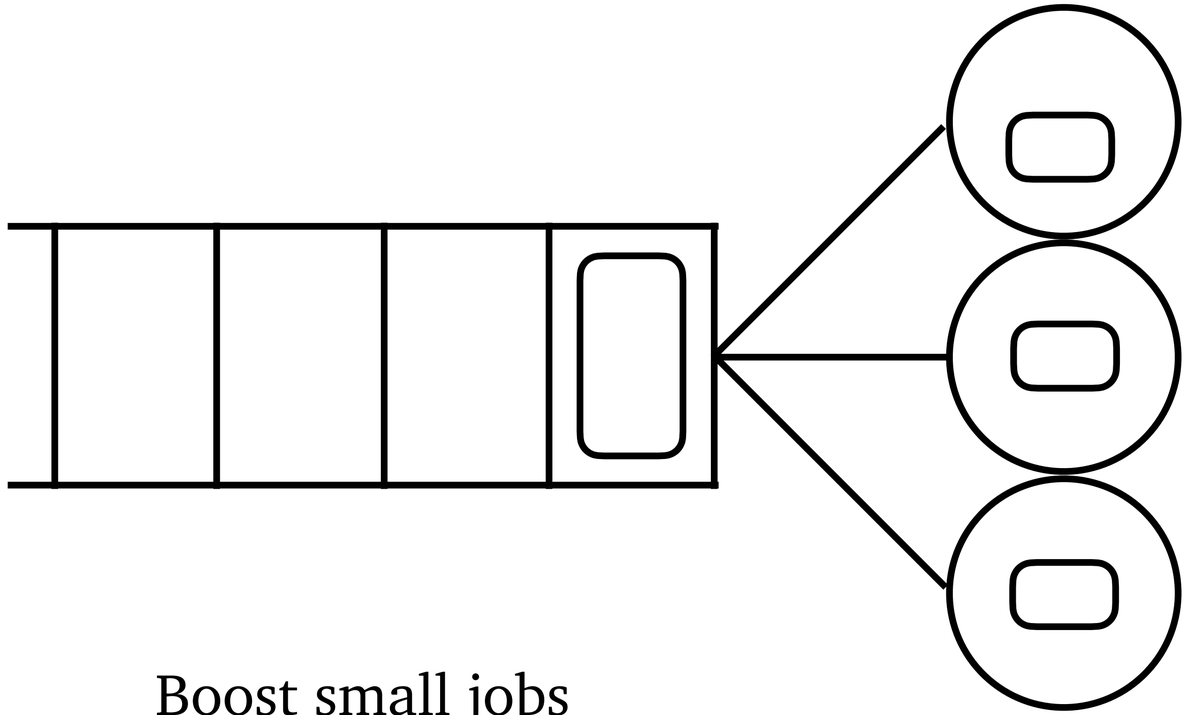

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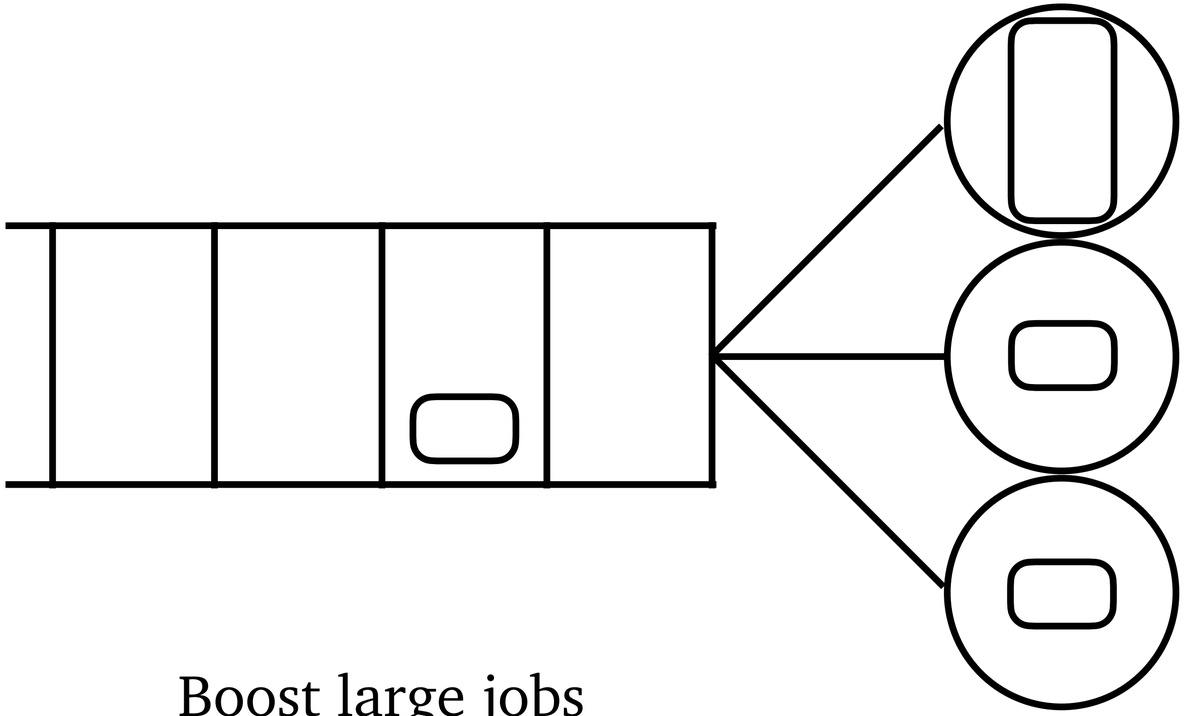

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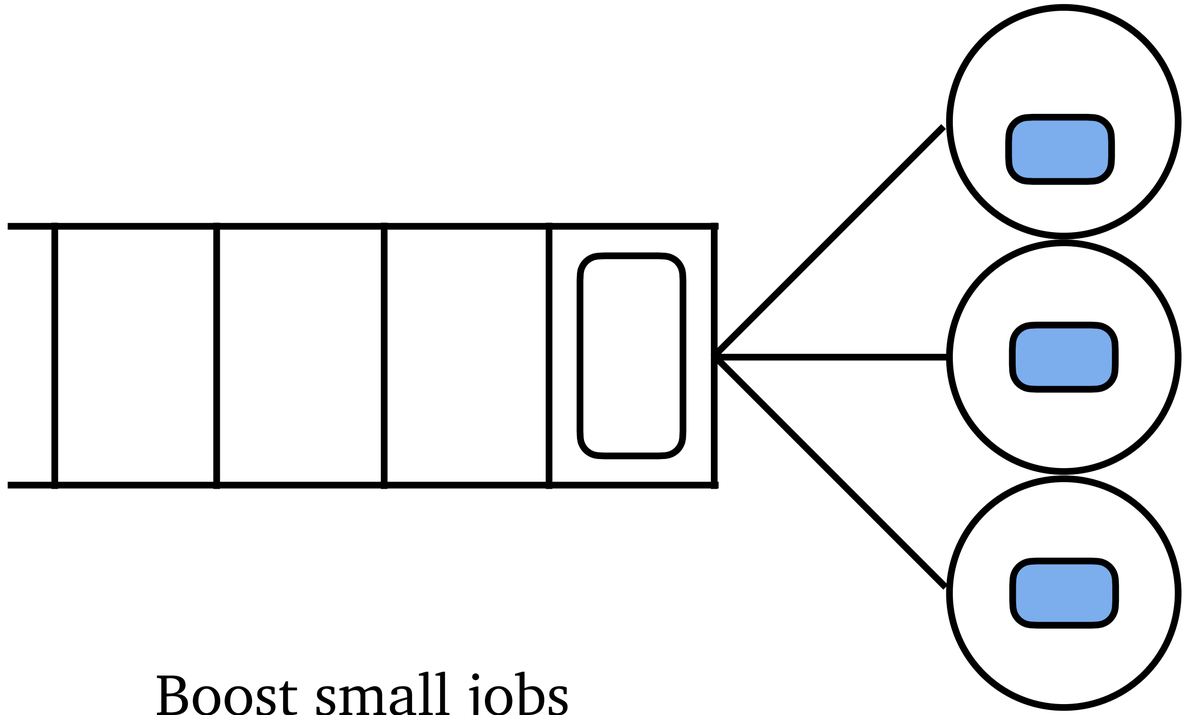

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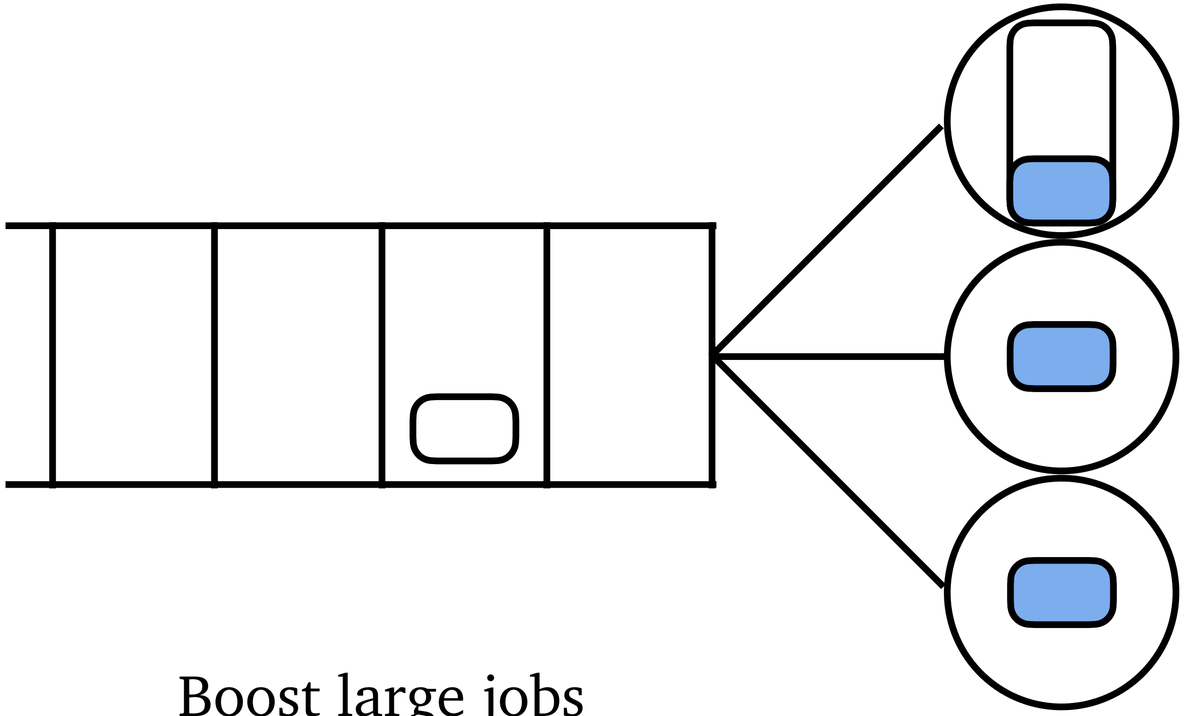

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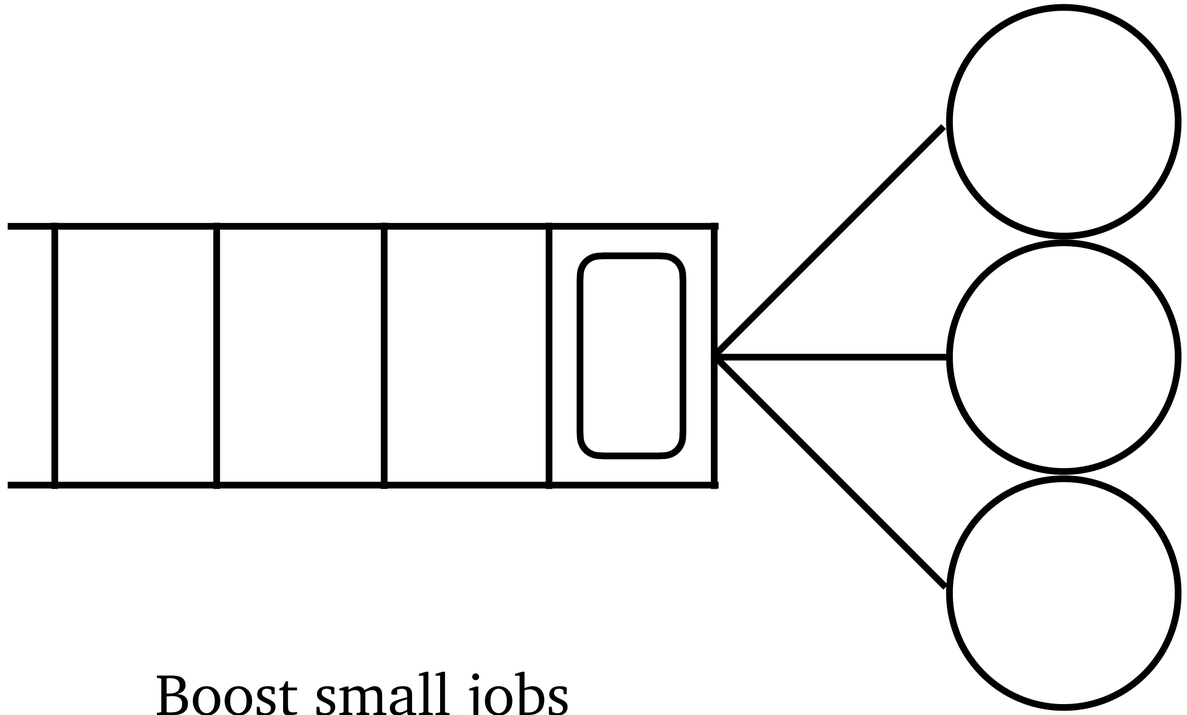

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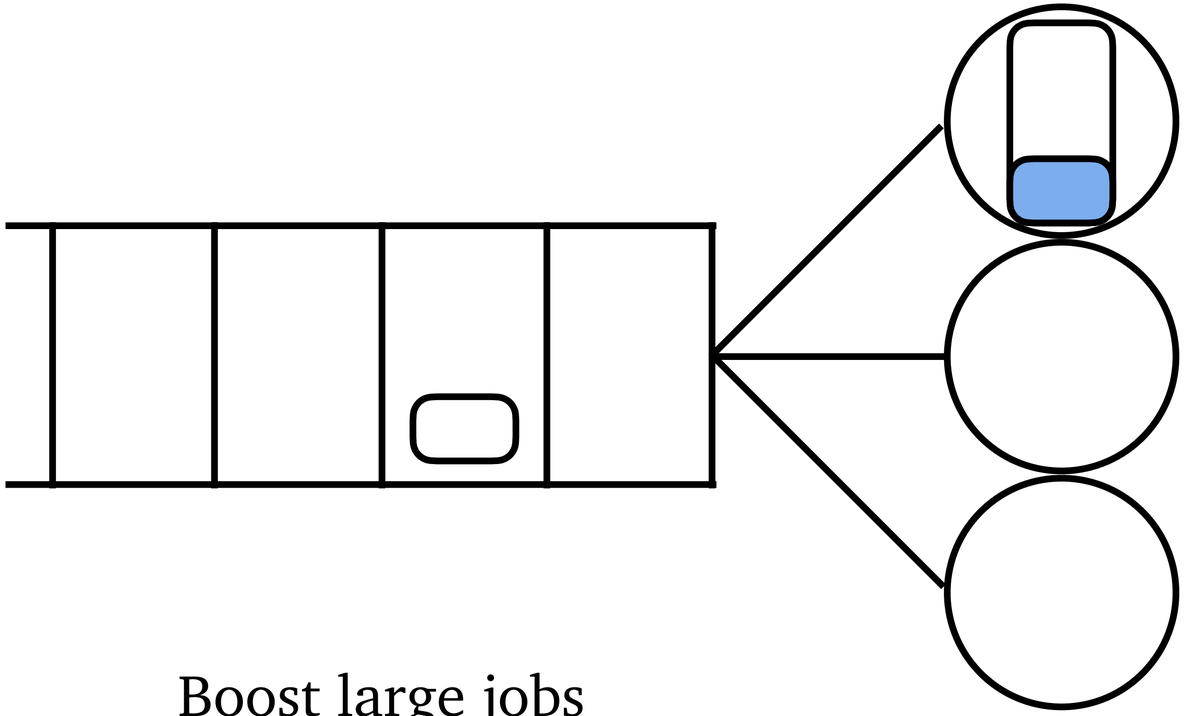

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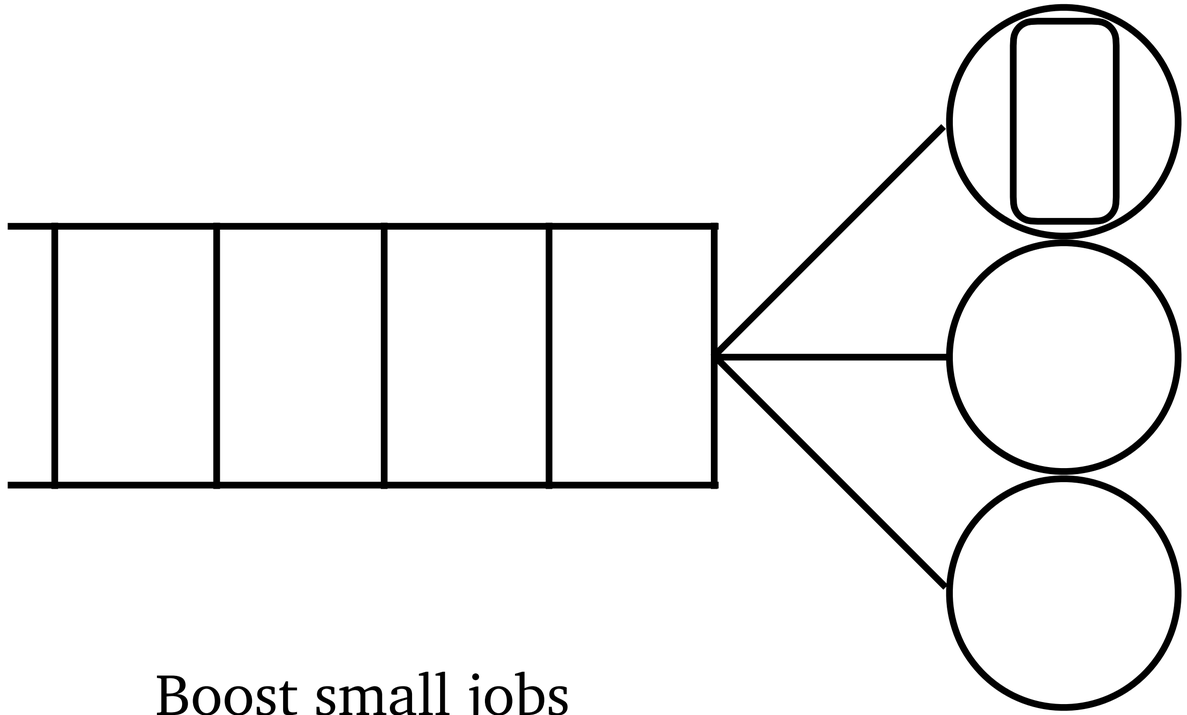

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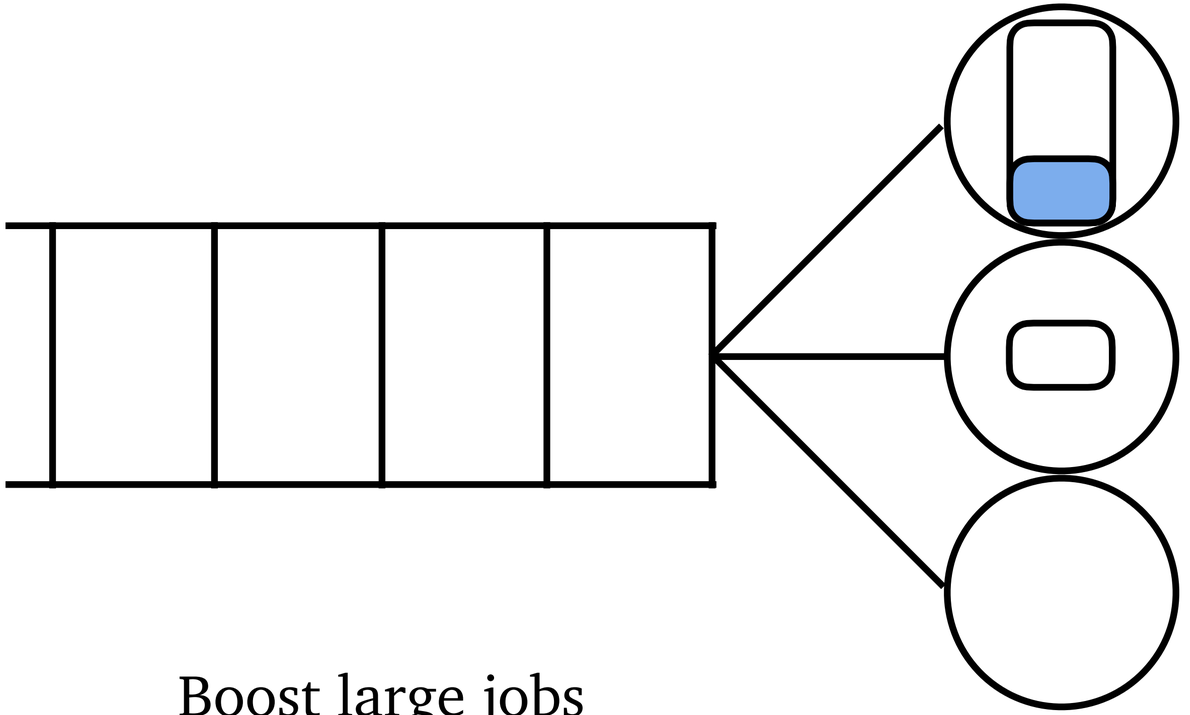

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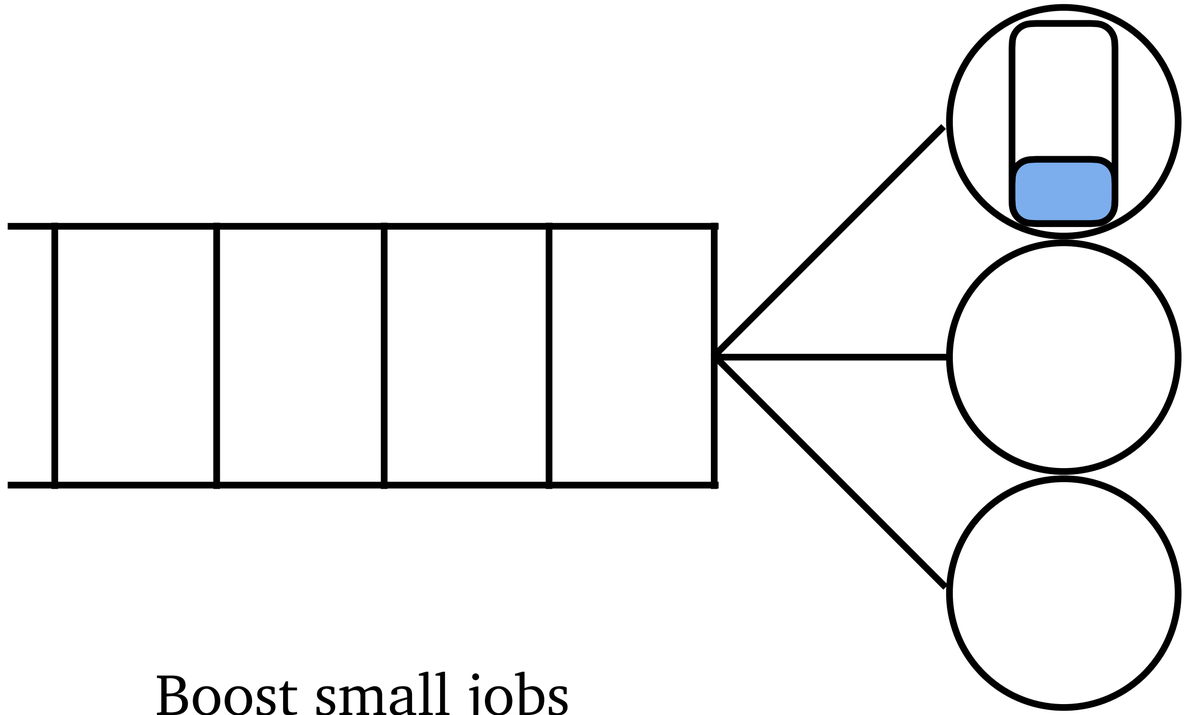

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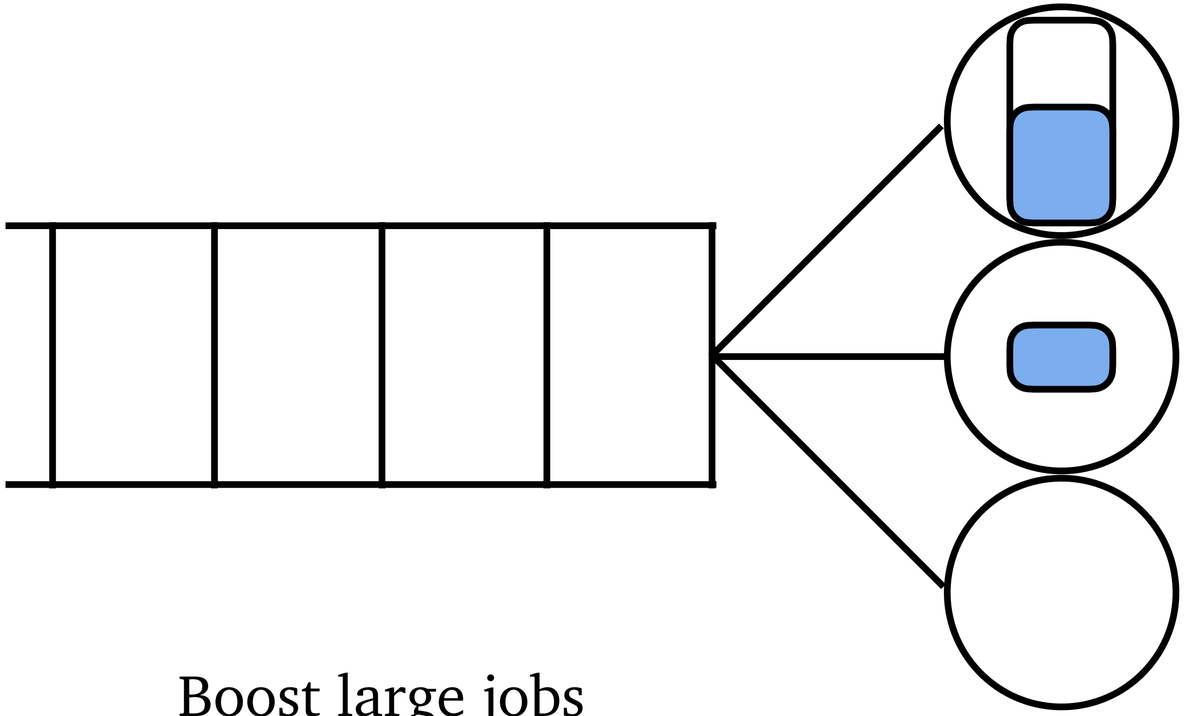

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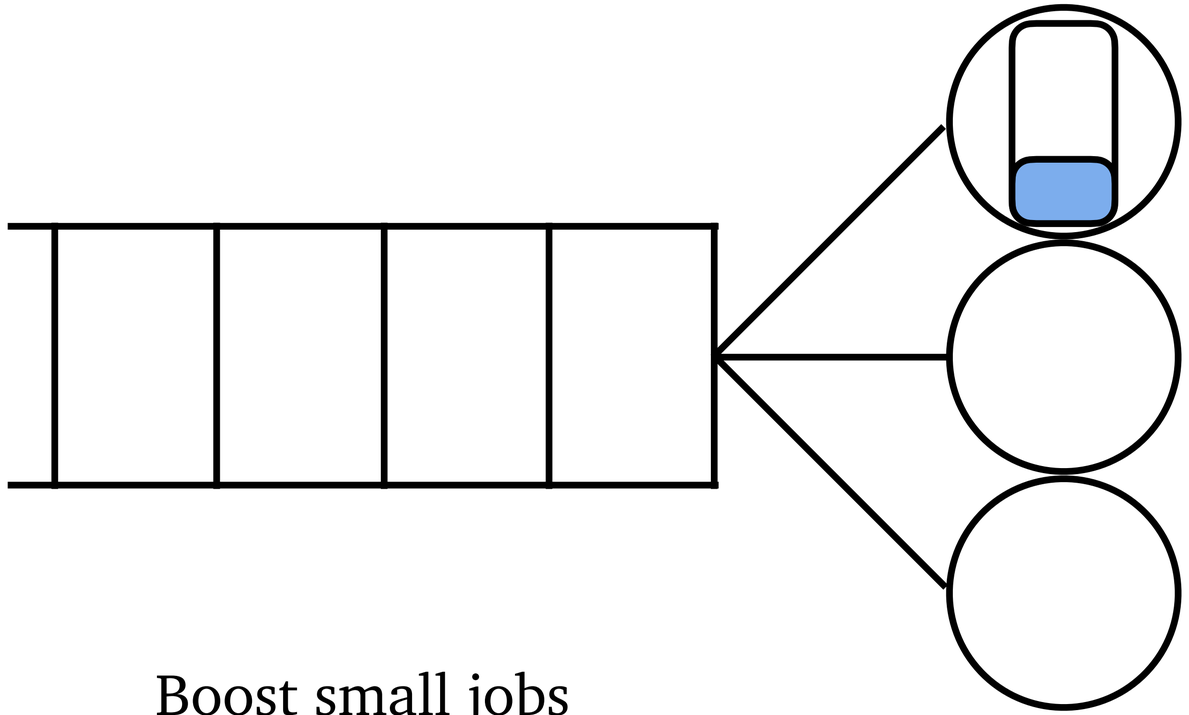

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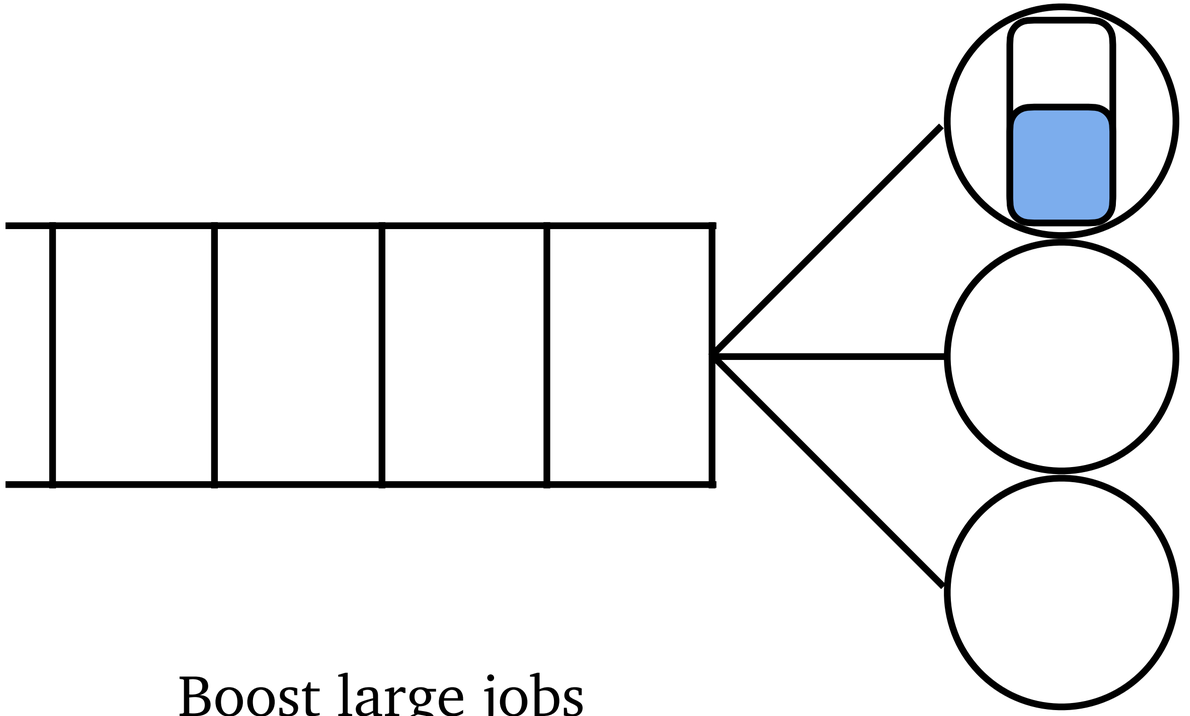

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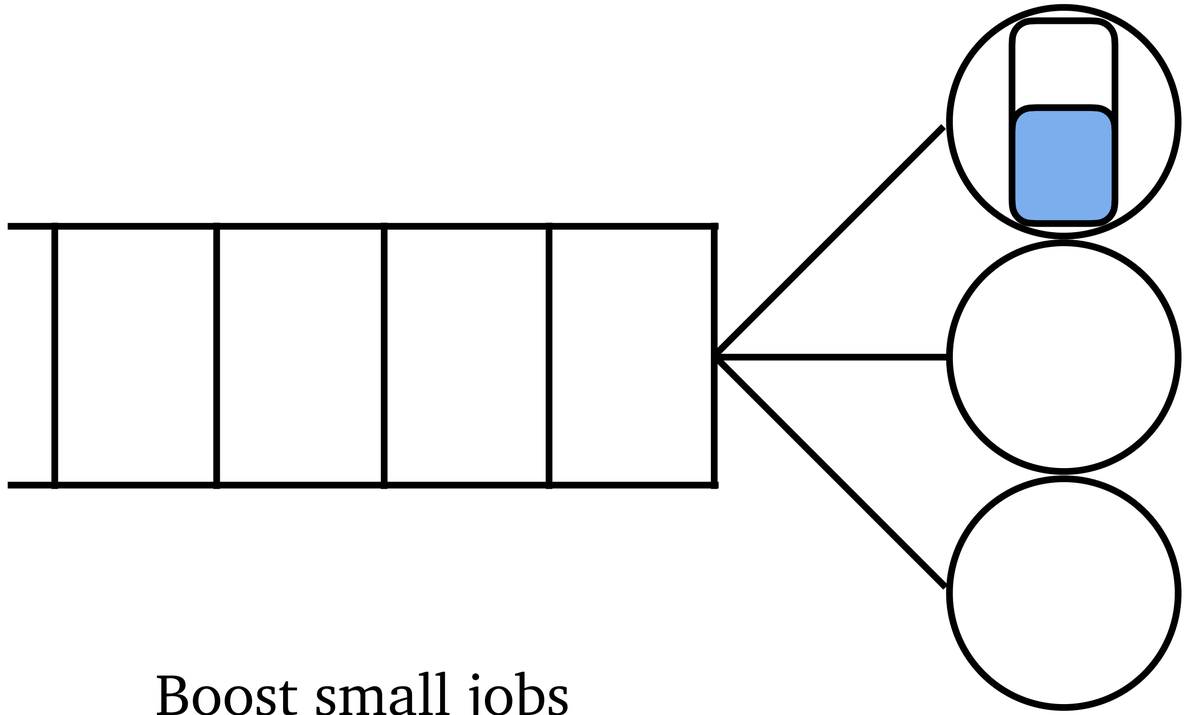

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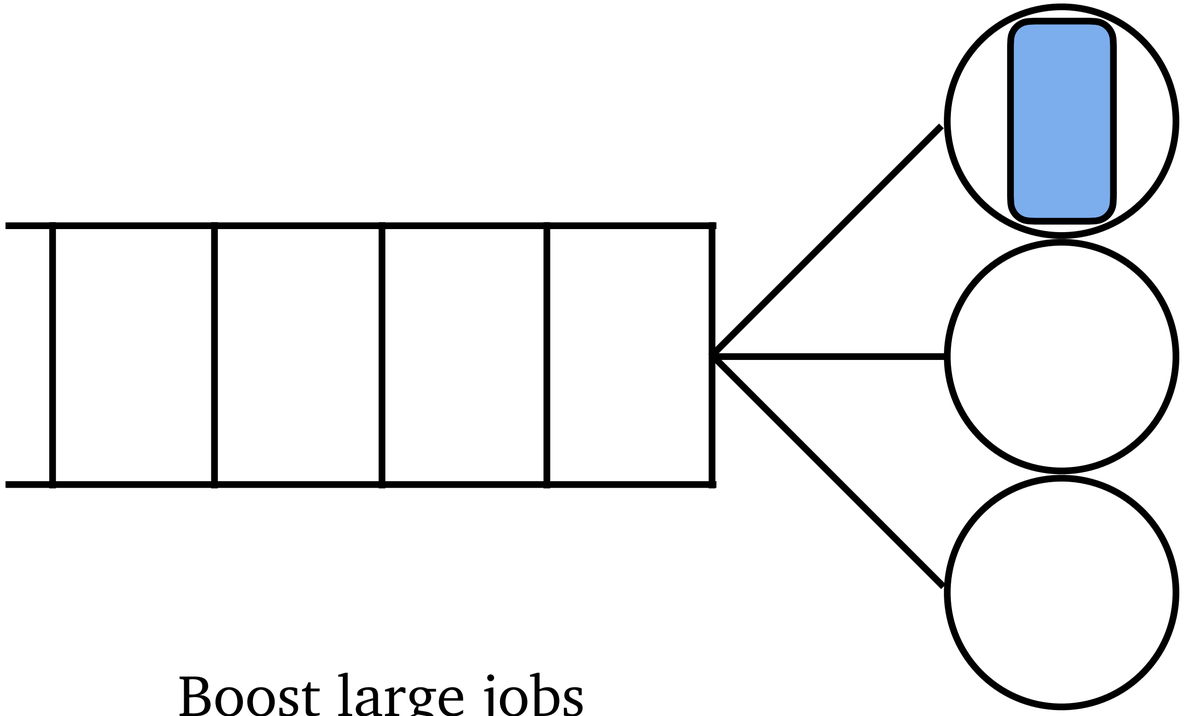

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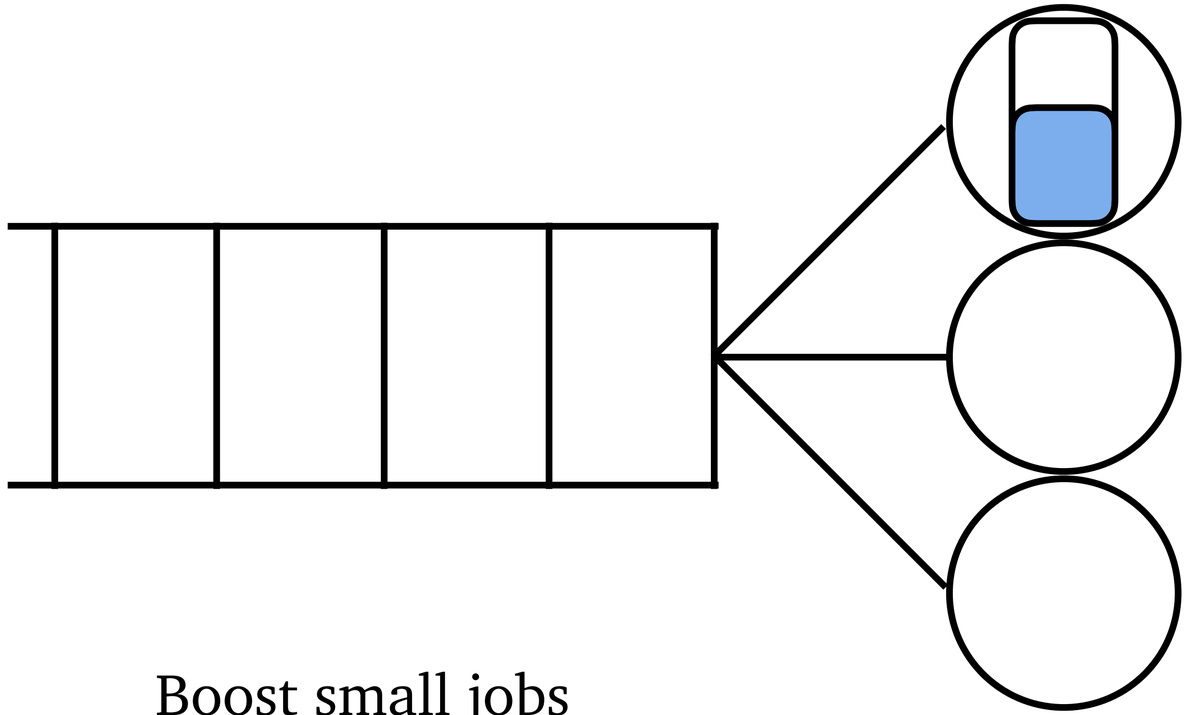

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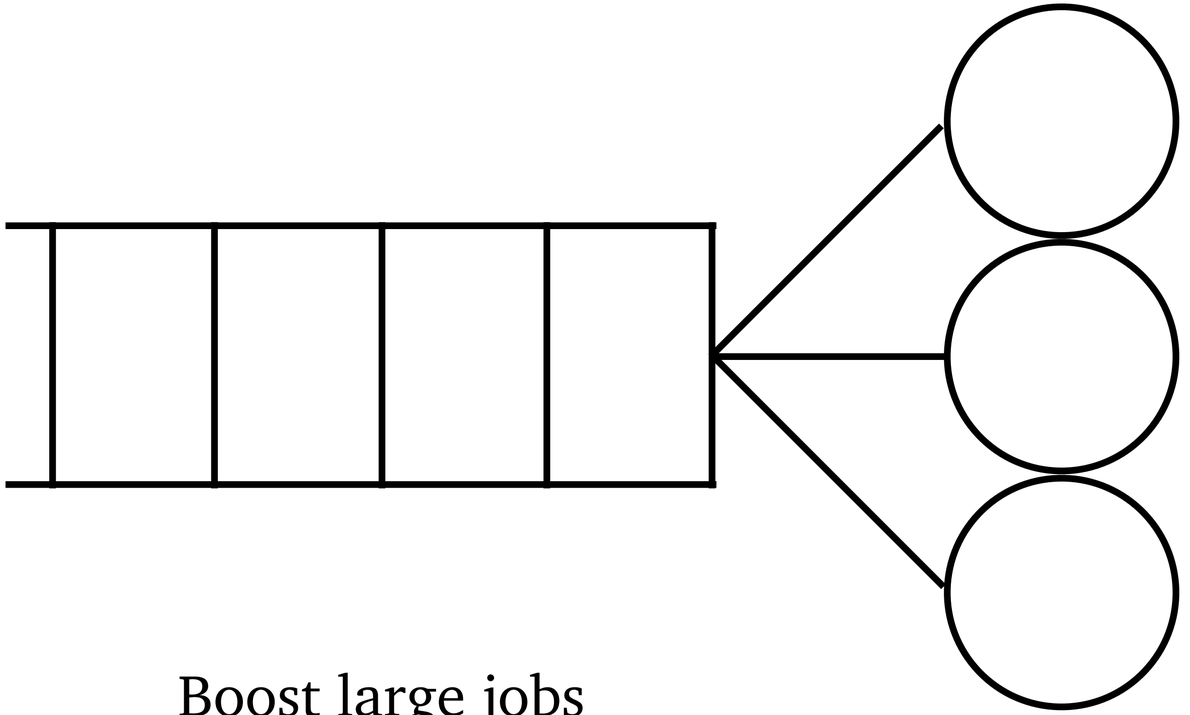

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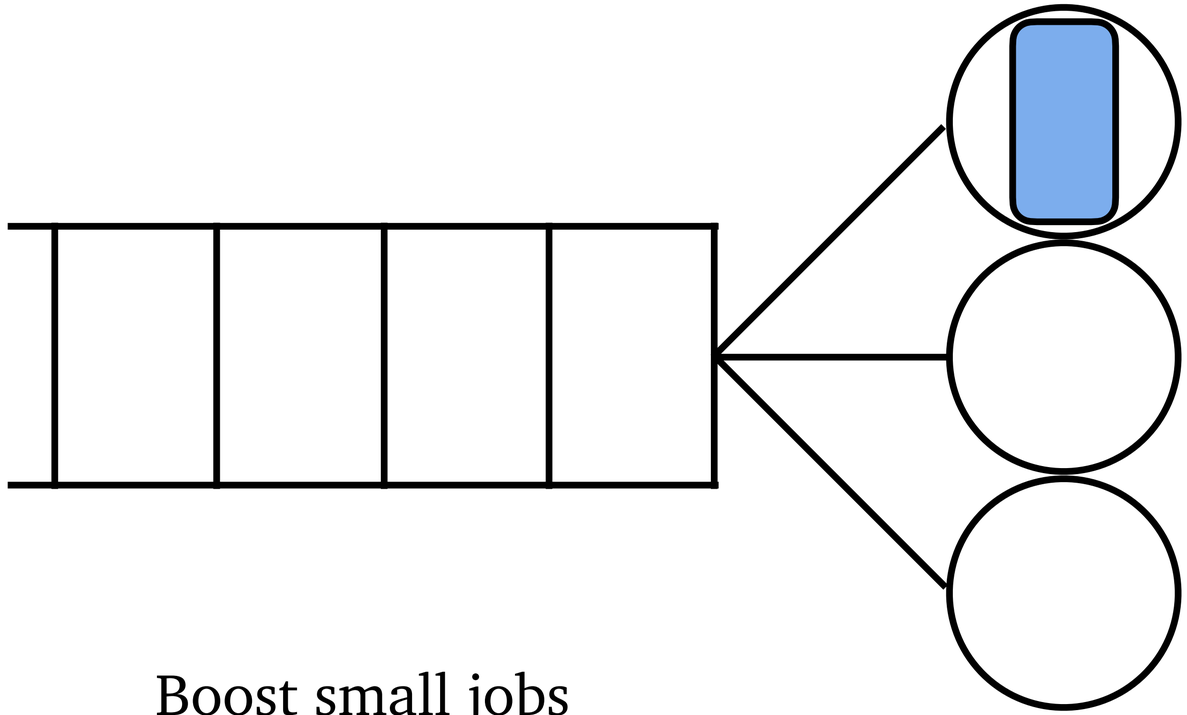

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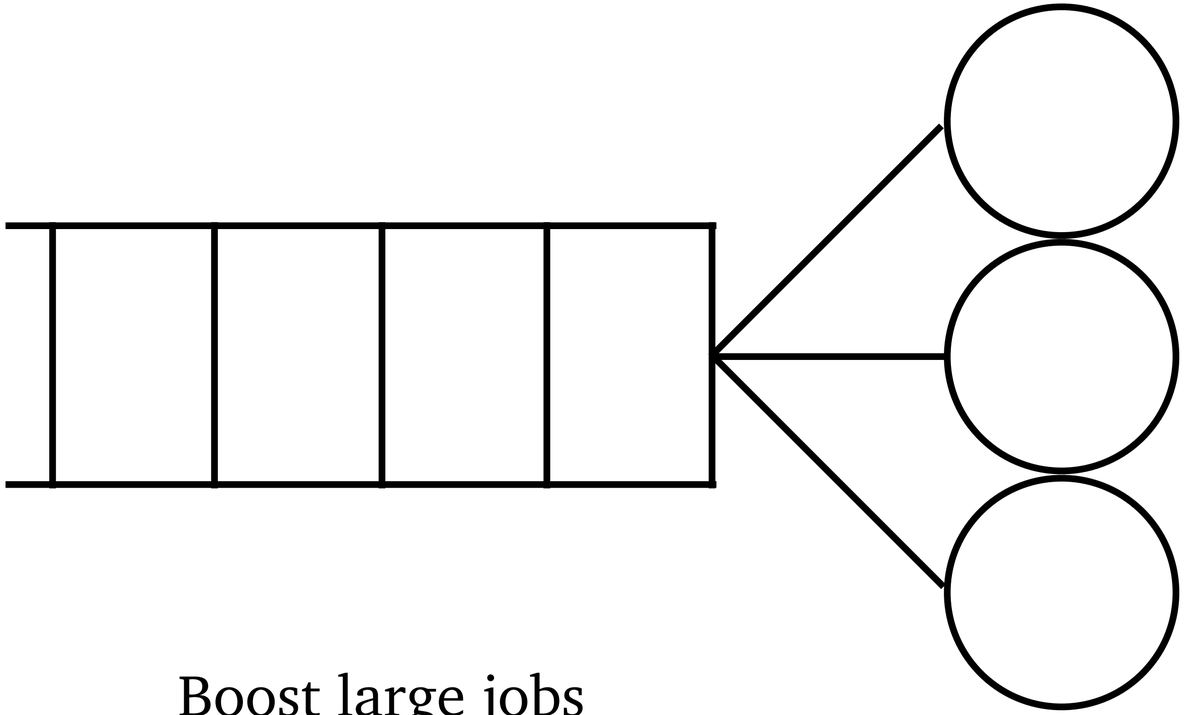

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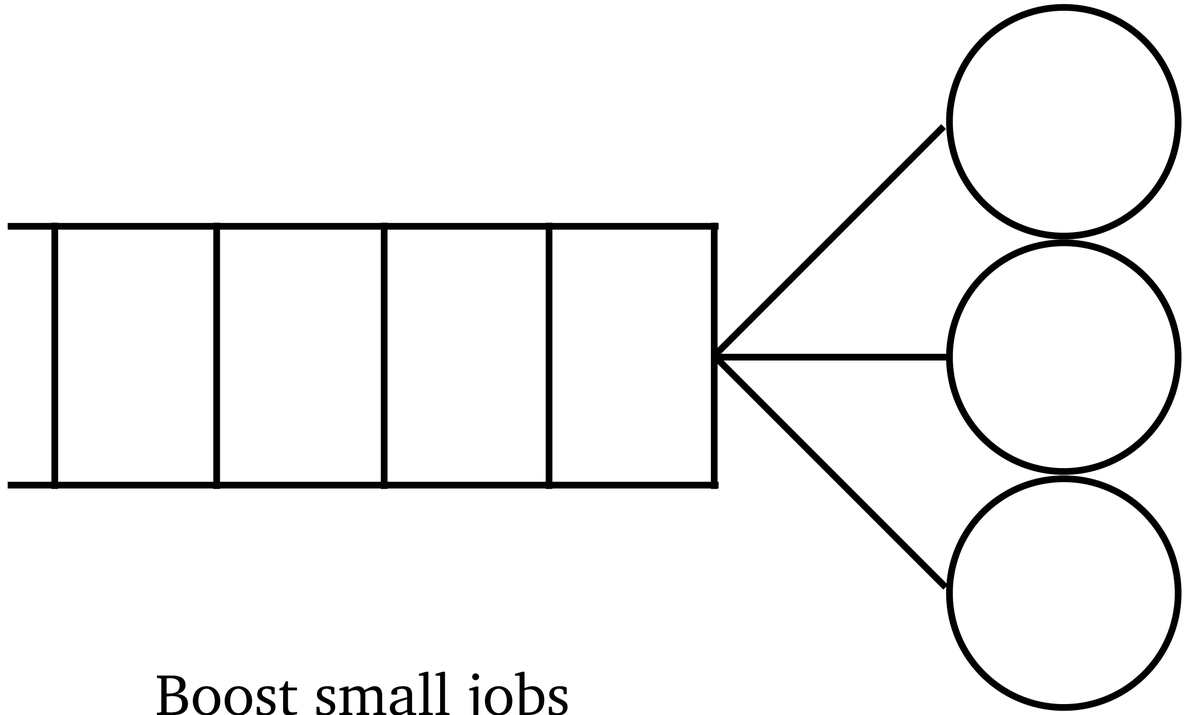

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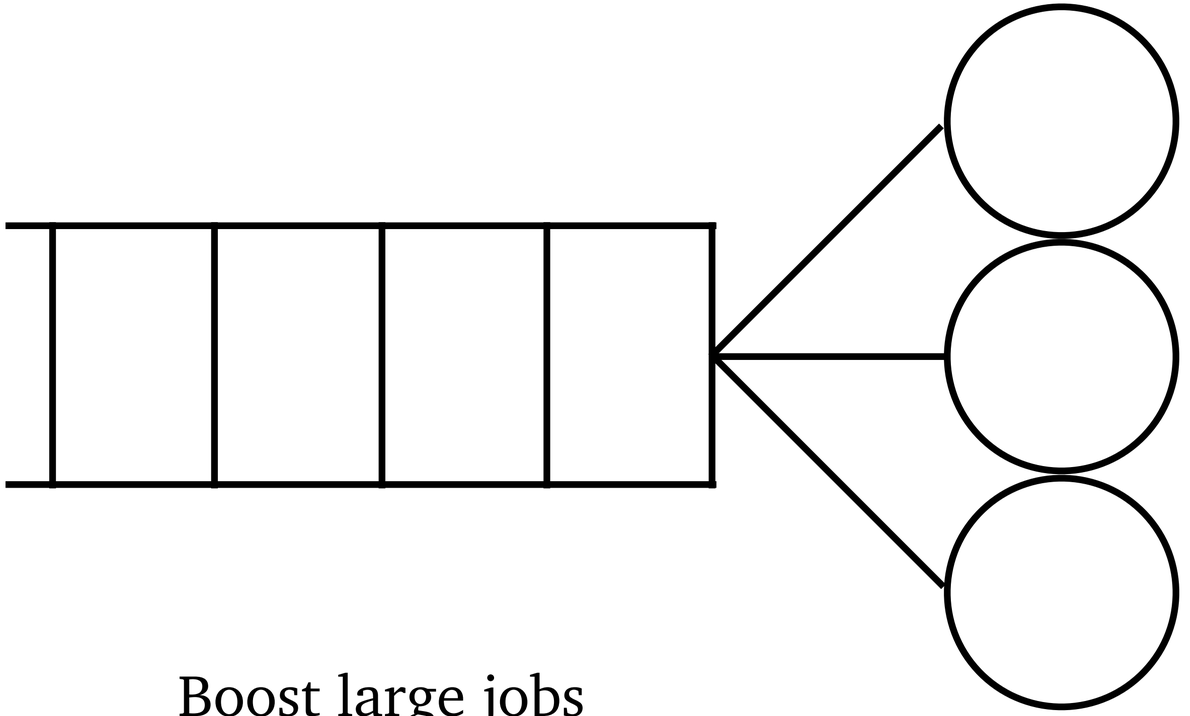
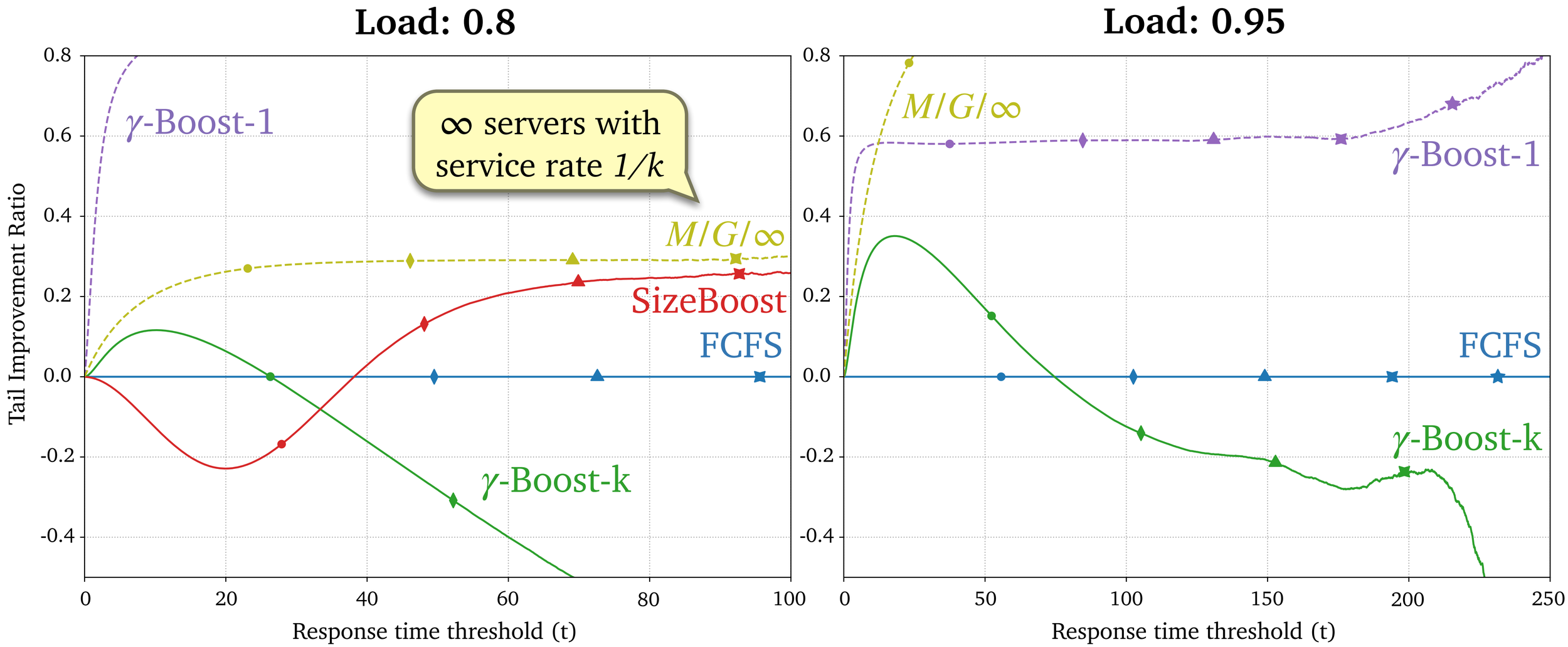


Boost large jobs

γ-Boost performance in lighter traffic



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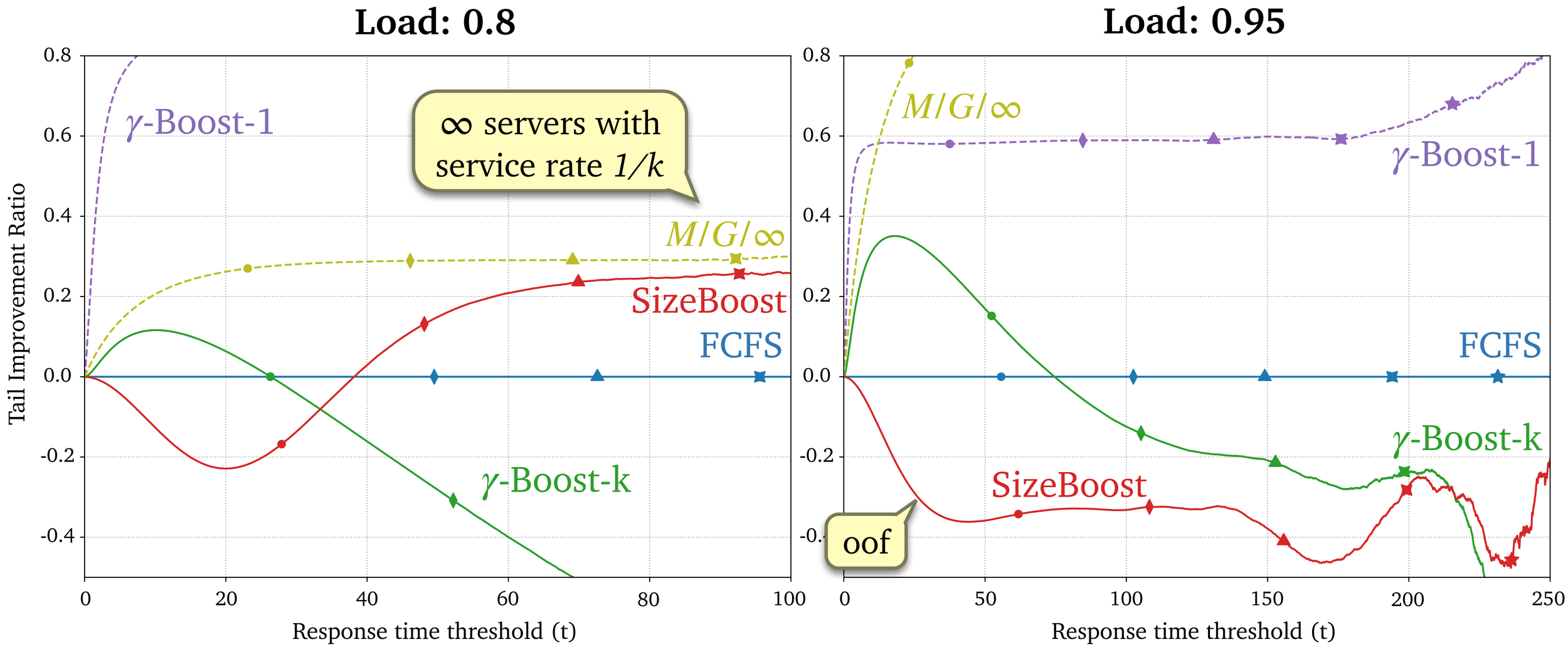

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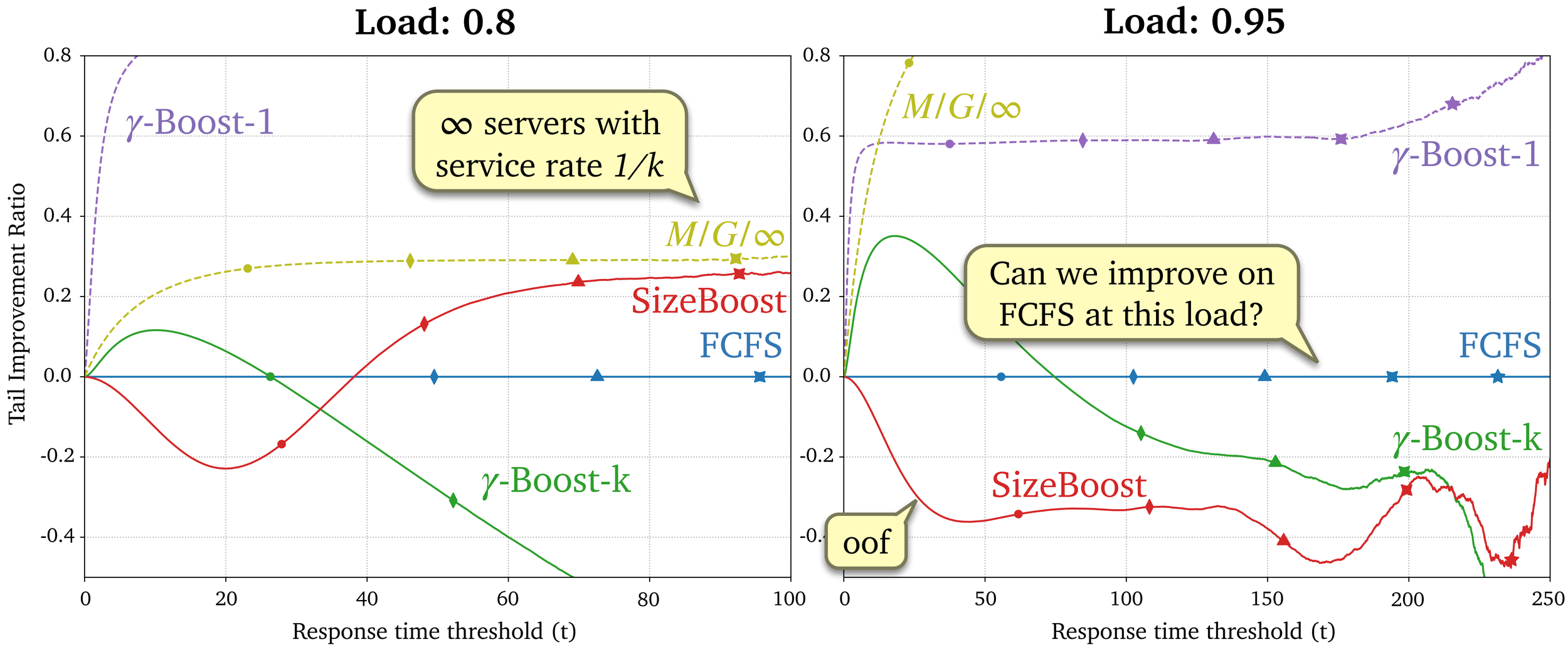

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90th percentile · 99th percentile · 99.9th percentile · 99.99th percentile · 99.999th percentile

Where does γ-Boost come from?

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Queueing Problem

$$\text{minimize} \quad C_\pi = \lim_{t\to\infty} e^{\gamma t}\,\mathbf{P}[T_\pi > t] = \lim_{\theta\to\gamma} \frac{\gamma-\theta}{\gamma}\mathbf{E}[e^{\theta T_\pi}]$$

(by final value theorem for Laplace transforms)

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Batch Problem

$$\text{minimize } \frac{1}{n}\sum_{i=1}^{n} e^{\gamma t_i}$$

minimized by γ-Boost in M/G/1

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Queueing Problem

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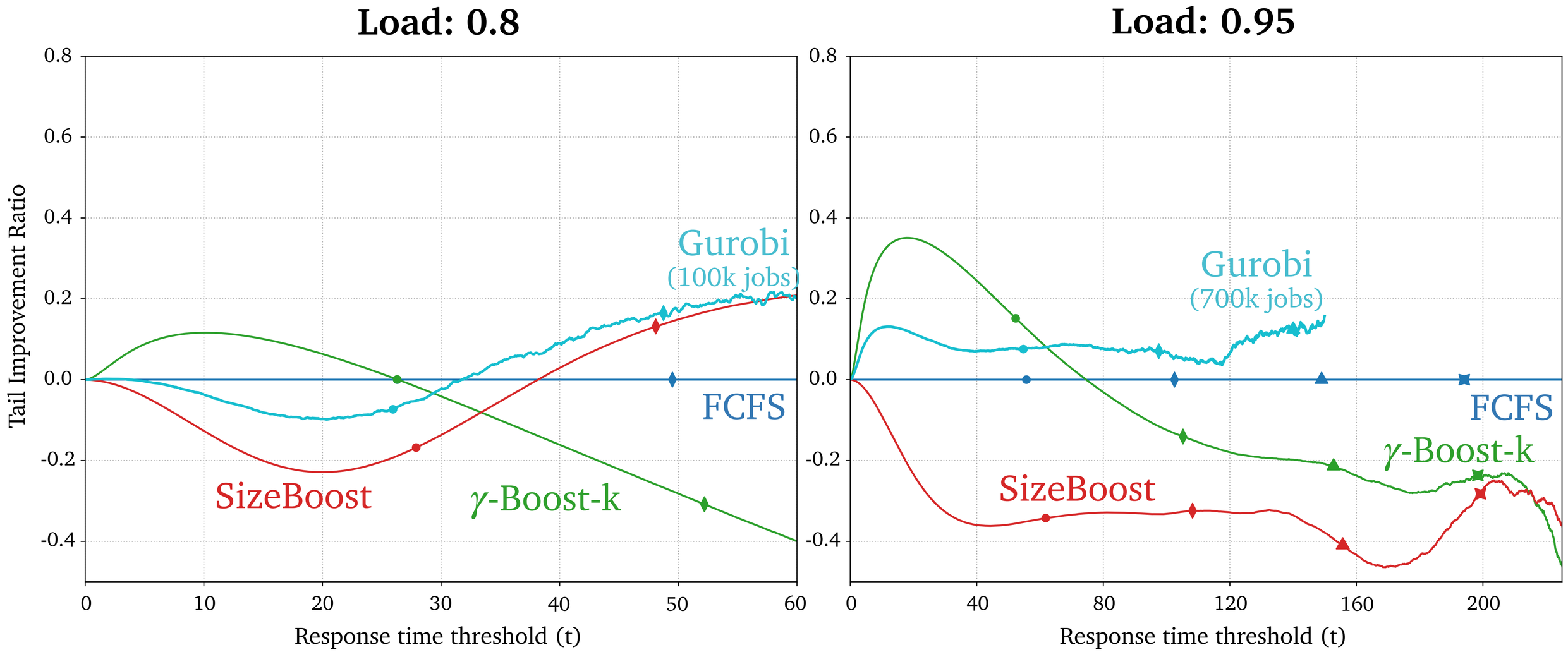
minimized by γ-Boost in M/G/1

Numerically optimize $\frac{1}{n}\sum_{i=1}^{n} e^{\gamma t_i}$ in M/G/k after each arrival.

Numerically optimizing the batch problem in the M/G/k



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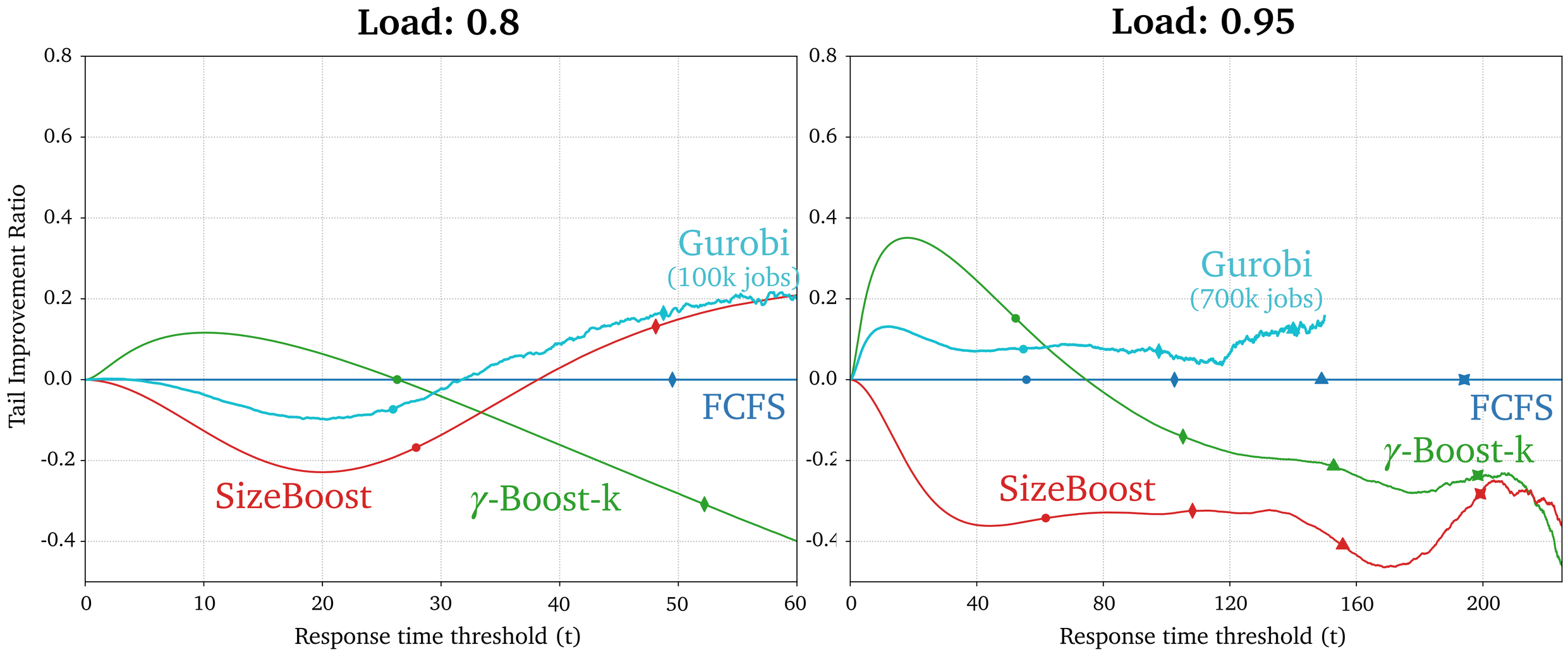
One week before submission…



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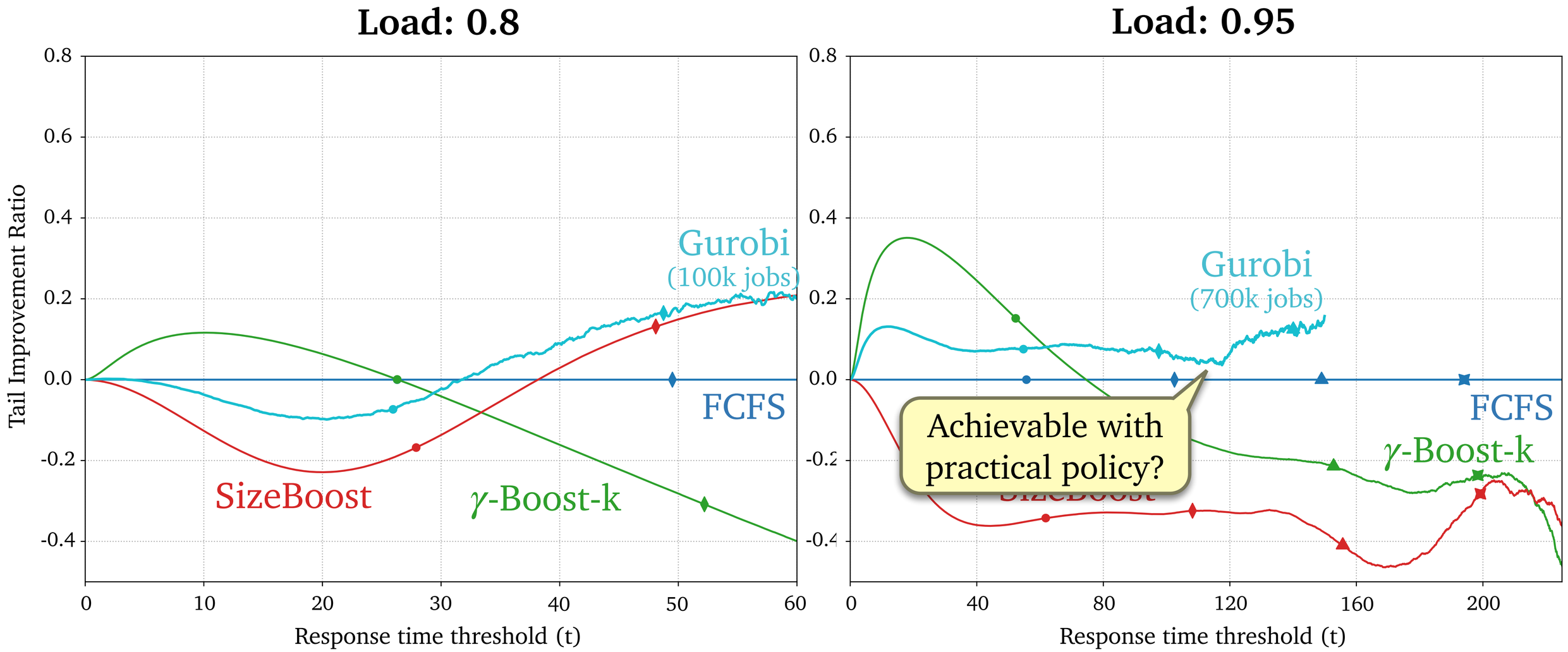

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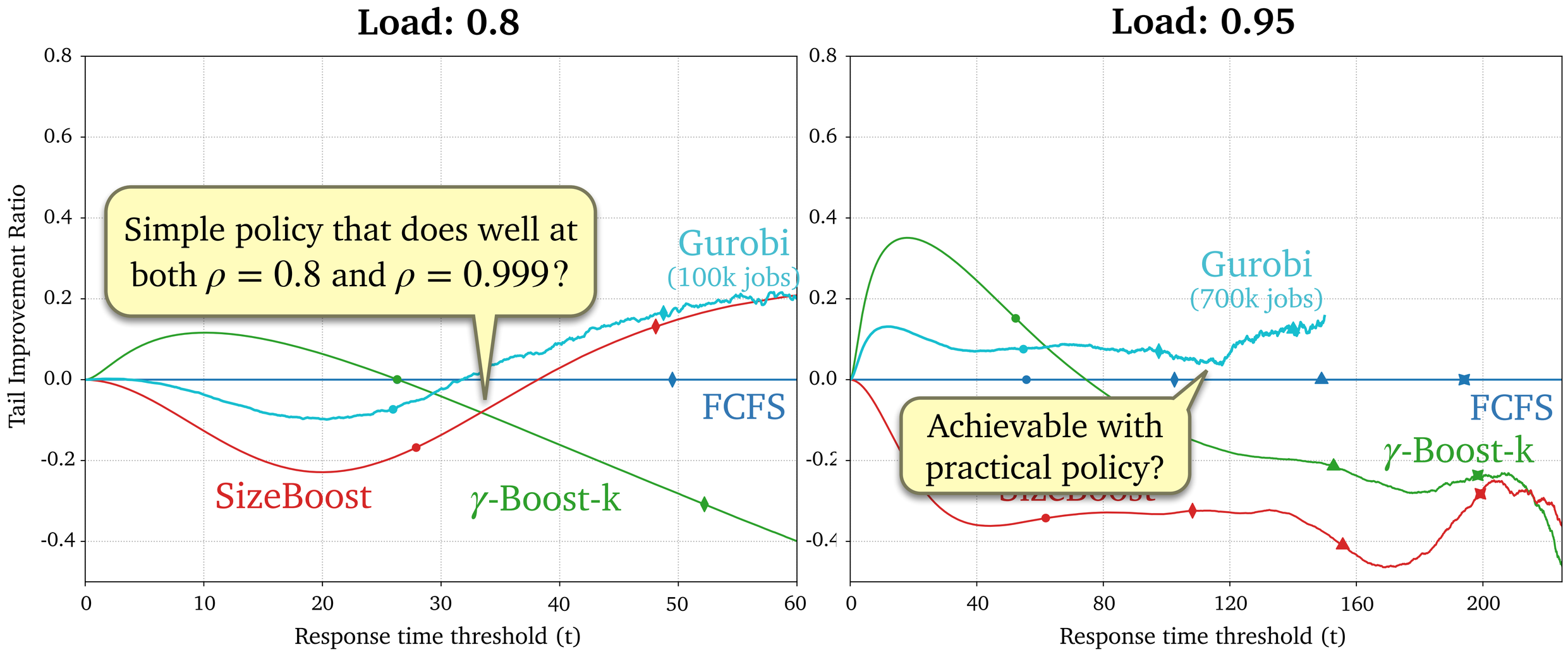

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Is there some way to combine γ-Boost and SizeBoost?

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Add the boosts from both policies together!

Why does adding the boosts make any sense?

10 servers,
Exp(1) job sizes

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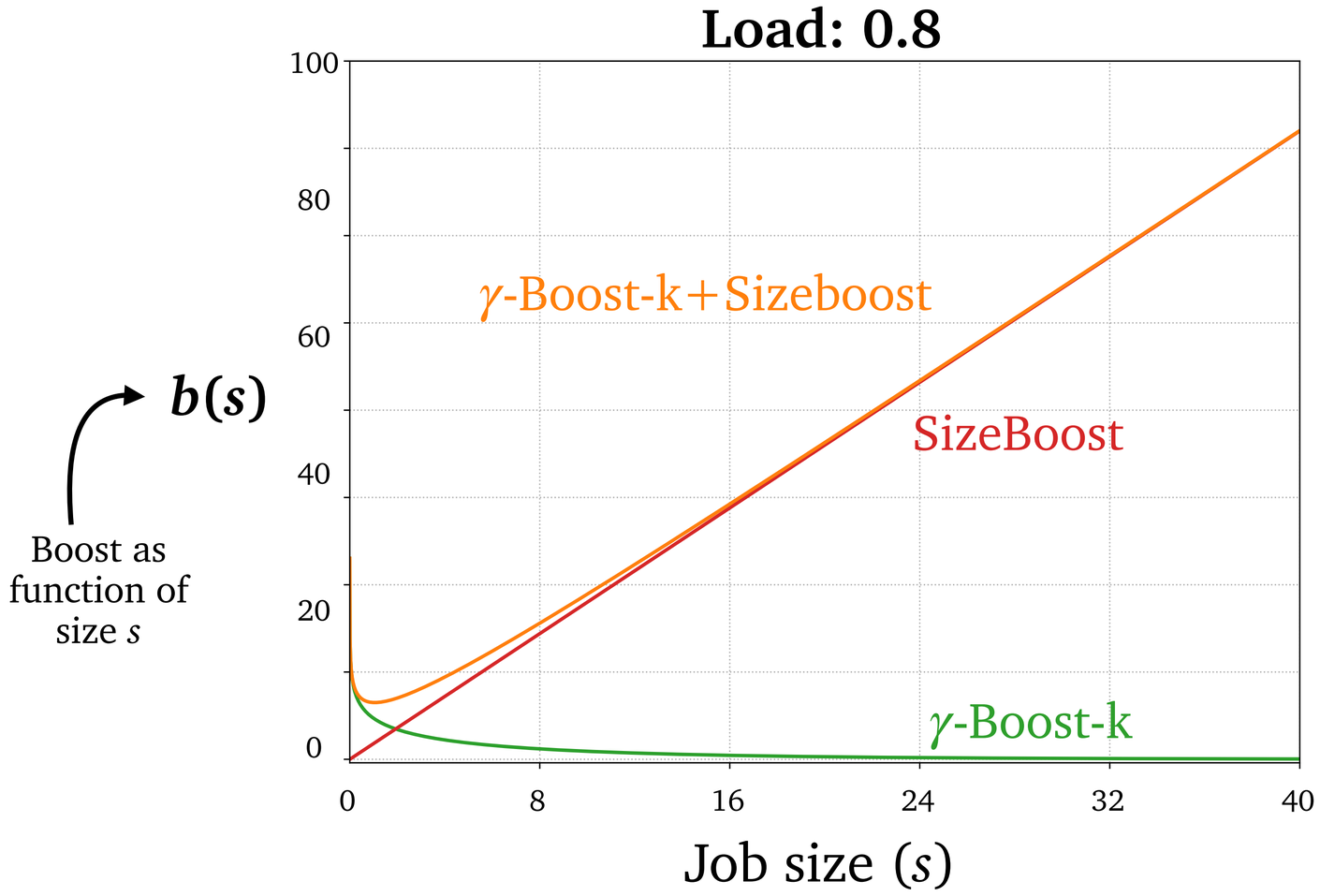

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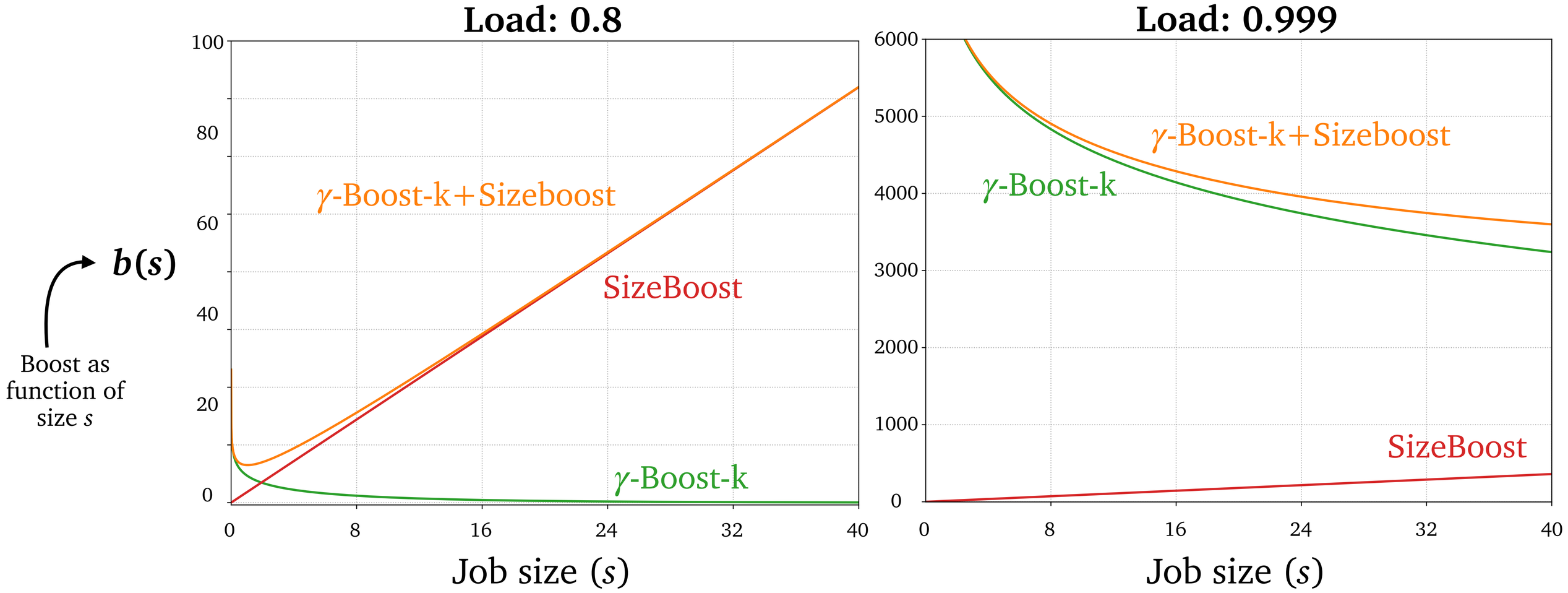

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Is this policy good at both $\rho = 0.8$ and $\rho = 0.999$?

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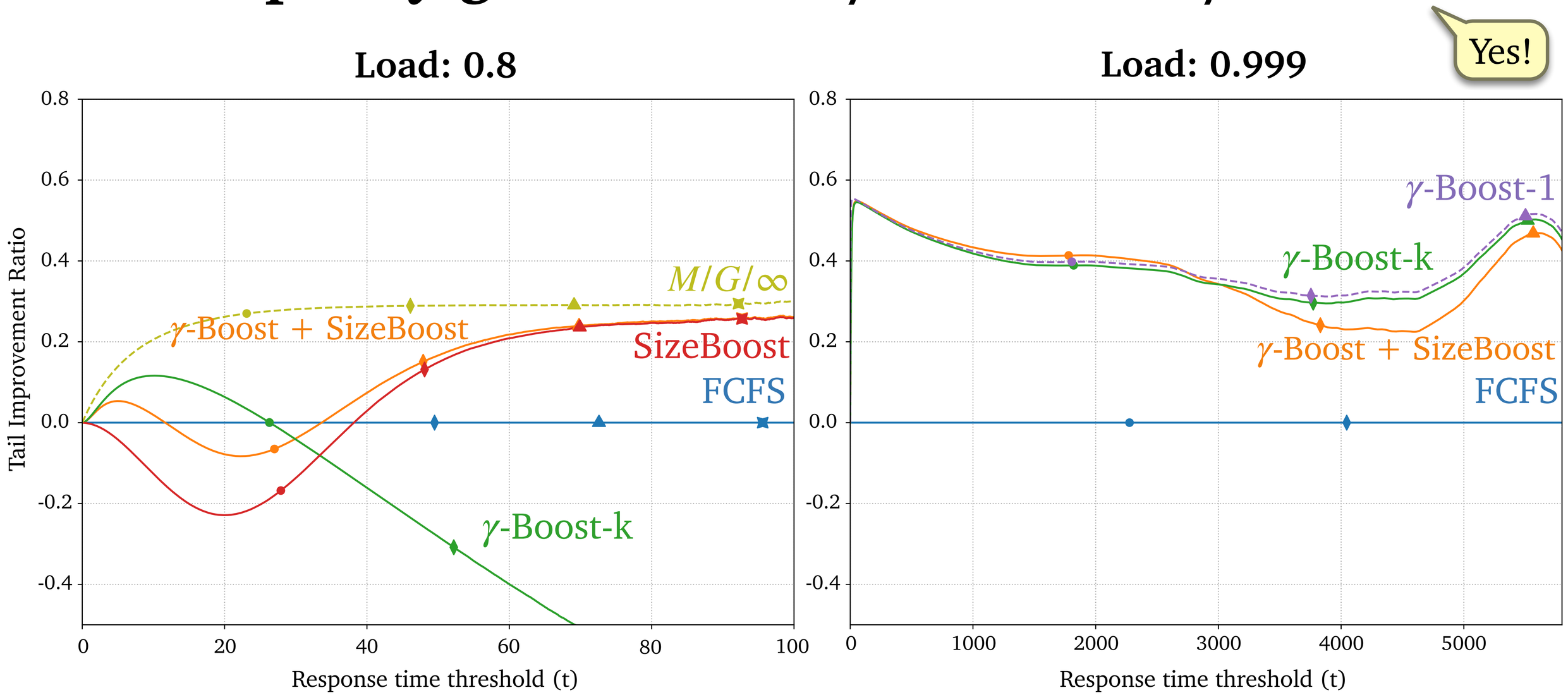
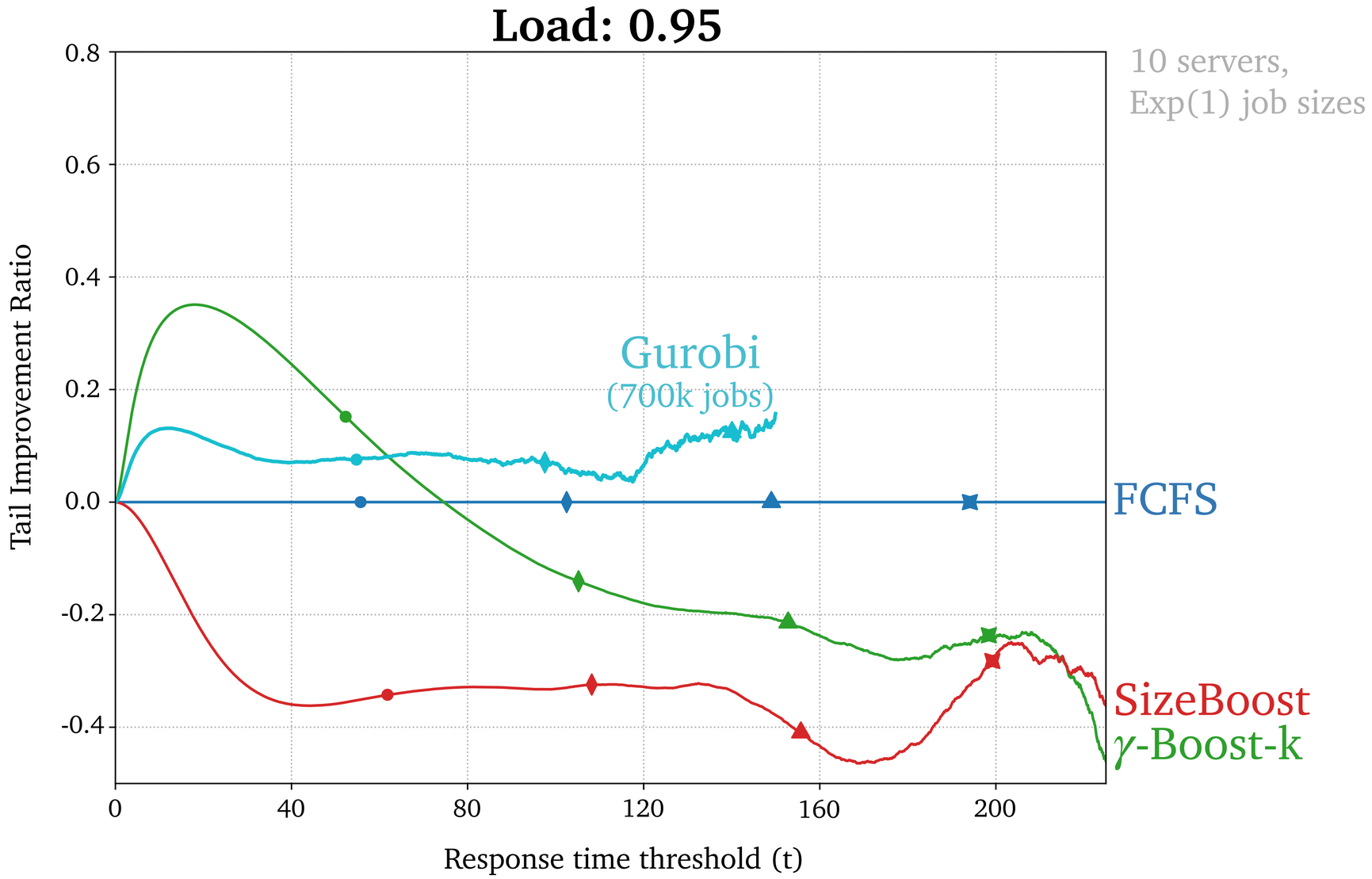


How does this policy perform at $\rho = 0.95$?



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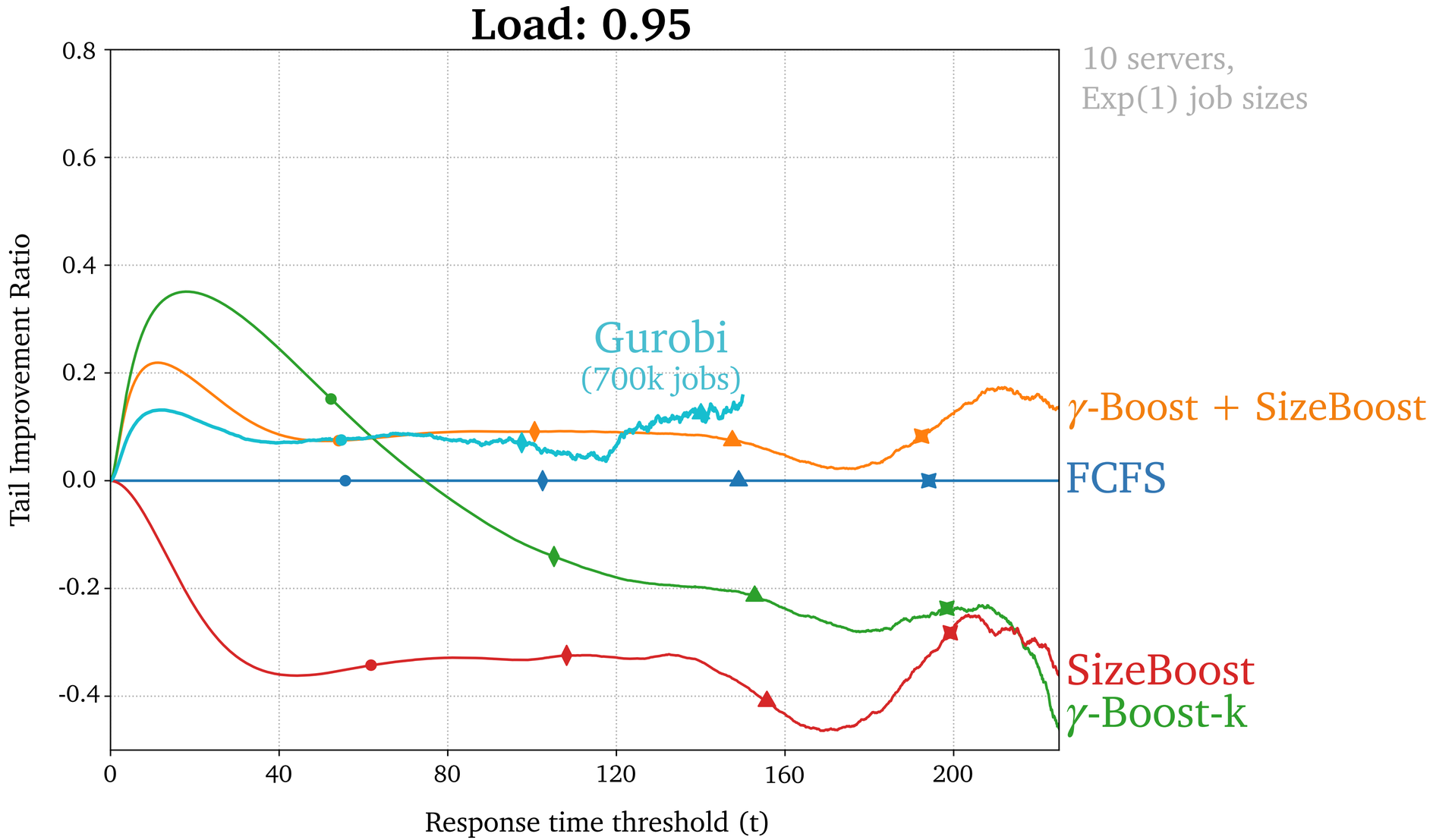
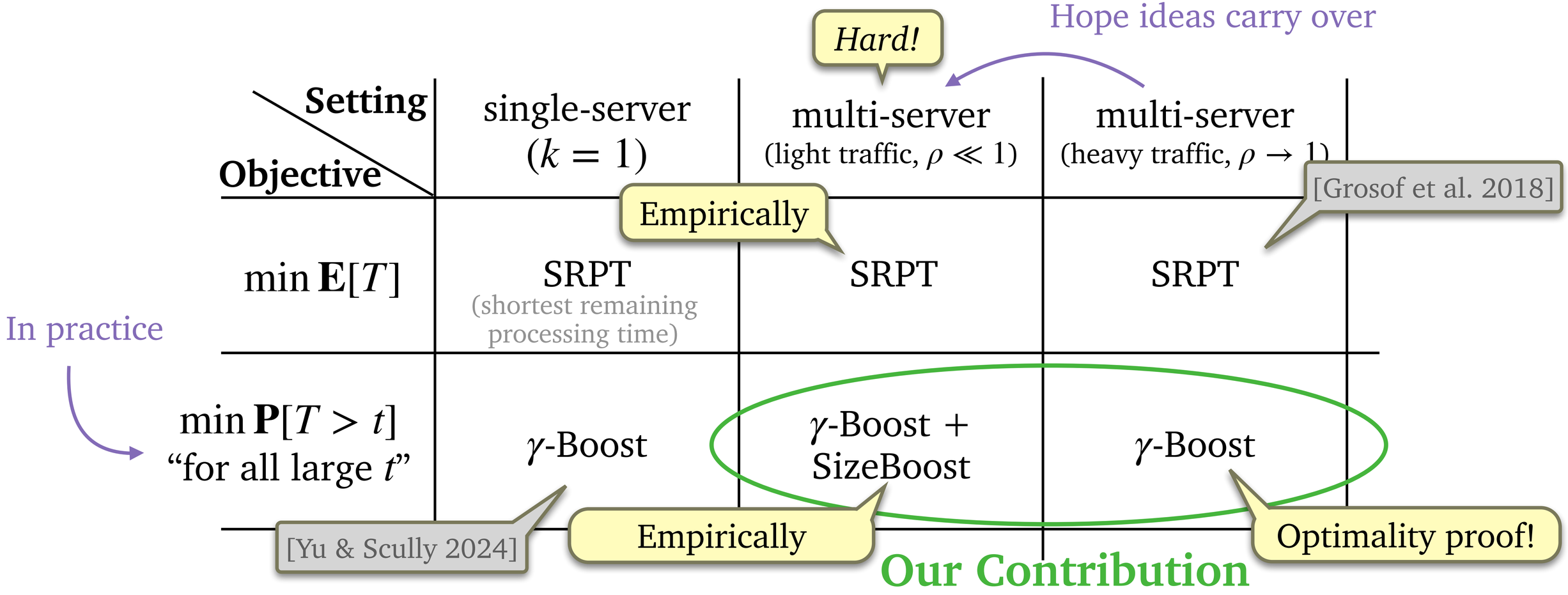
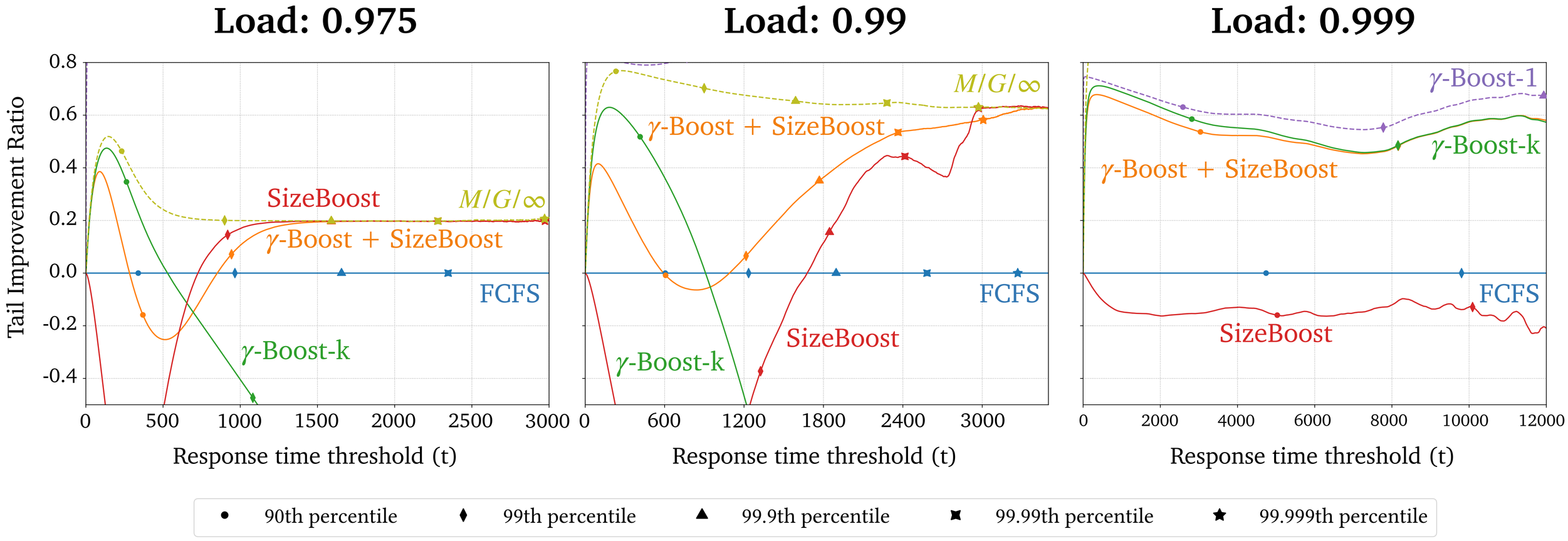


How does this [image] m at $\rho = 0.95$?

Summary


Hard!
Hope ideas carry over
Setting
Objective
single-server
$(k = 1)$
multi-server
(light traffic, $\rho \ll 1$)
multi-server
(heavy traffic, $\rho \to 1$)
Empirically
[Grosof et al. 2018]
$\min \mathbf{E}[T]$
SRPT
(shortest remaining processing time)
SRPT
SRPT
In practice
$\min \mathbf{P}[T > t]$
"for all large t"
γ-Boost
γ-Boost + SizeBoost
γ-Boost
[Yu & Scully 2024]
Empirically
Optimality proof!
Our Contribution

100 servers



100 servers,
Hyperexponential job sizes